Recent Developments in the Modeling of Heavy Quarkonia

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Abstract. We examine the spectra and radiative decays of the $c\bar{c}$ and $b\bar{b}$ systems using a model which incorporates the complete one-loop spin-dependent perturbative QCD short distance potential, a linear confining term including (spin-dependent) relativistic corrections to order v^2/c^2, and a fully relativistic treatment of the kinetic energy. We compare the predictions of this model to experiments, including states and decays recently measured at Belle, BaBar, CLEO and CDF.

PACS: 12.39.Pn,14.40.Lb,14.40.Nd

1. INTRODUCTION AND THE POTENTIAL MODEL

Over the past 25+ years, potential models have proven valuable in analyzing the spectra and characteristics of heavy quarkonium systems [1, 2, 3, 4, 5, 6, 7]. Motivation for revisiting the potential model interpretation of the $c\bar{c}$ and $b\bar{b}$ systems is provided by recent experimental results:

- The discovery of several expected states in the charmonium spectrum (η_C and h_C)
- The discovery of new states [$X(3872)$, $X(3943)$], which could be a interpreted as above threshold charmonium levels
- The discovery of the 1^3D_2 state of the upsilon system
- The determination of various $E1$ widths for $c\bar{c}$ and $b\bar{b}$.

Our purpose here is to examine to what extent a semi-relativistic potential model which includes all v^2/c^2 and one-loop QCD corrections can fit the below threshold $c\bar{c}$ and $b\bar{b}$ data and accommodate the new above threshold states.

In our analysis, we use a semi-relativistic Hamiltonian of the form

$$\begin{aligned} H &= 2\sqrt{\vec{p}^2+m^2}+Ar-\frac{4\alpha_S}{3r}\left(1+\frac{\alpha_S}{3\pi}(12-n_f)(\ln(\mu r)+\gamma_E)\right)+V_S+V_L \\ &= H_0+V_S+V_L\,, \end{aligned}$$

where μ is the renormalization scale, V_L contains the v^2/c^2 corrections to the confining potential and the short distance potential is $V_S = V_{HF}+V_{LS}+V_T+V_{SI}$, with

$$\begin{aligned} V_{HF} = \; & \frac{32\pi\alpha_S\vec{S}_1\cdot\vec{S}_2}{9m^2}\left\{\left[1-\frac{\alpha_S}{12\pi}(26+9\ln 2)\right]\delta(\vec{r})\right. \\ & \left. -\frac{\alpha_S}{24\pi^2}(33-2n_f)\nabla^2\left[\frac{\ln\mu r+\gamma_E}{r}\right]+\frac{21\alpha_S}{16\pi^2}\nabla^2\left[\frac{\ln mr+\gamma_E}{r}\right]\right\} \end{aligned}$$

$$V_{LS} = \frac{2\alpha_S \vec{L}\cdot\vec{S}}{m^2 r^3}\left\{1-\frac{\alpha_S}{6\pi}\left[\frac{11}{3}-(33-2n_f)\ln\mu r+12\ln mr-(21-2n_f)(\gamma_E-1)\right]\right\}$$

$$V_T = \frac{4\alpha_S(3\vec{S}_1\cdot\hat{r}\vec{S}_2\cdot\hat{r}-\vec{S}_1\cdot\vec{S}_2)}{3m^2r^3}\left\{1+\frac{\alpha_S}{6\pi}\left[8+(33-2n_f)\left(\ln\mu r+\gamma_E-\frac{4}{3}\right)\right.\right.$$
$$\left.\left.-18\left(\ln mr+\gamma_E-\frac{4}{3}\right)\right]\right\}$$

$$V_{SI} = \frac{4\pi\alpha_S}{3m^2}\left\{\left[1-\frac{\alpha_S}{2\pi}(1+\ln 2)\right]\delta(\vec{r})-\frac{\alpha_S}{24\pi^2}(33-2n_f)\nabla^2\left[\frac{\ln\mu r+\gamma_E}{r}\right]-\frac{7\alpha_S m}{6\pi r^2}\right\}$$

Using the variational procedure described in Ref. [8], we fit the experimental data for the the charm and Υ systems by varying the parameters A, α_S, m, μ and f_V, the fraction of vector coupling in the scalar-vector mixture of the confining potential, to find a minimum in χ^2. This was done in two ways: first by treating V_L+V_S as a perturbation and second by treating the entire Hamiltonian non-perturbatively. The results are shown in Table 1.

TABLE 1. Fitted Parameters

	$c\bar{c}$ Pert	$c\bar{c}$ Non-pert	$b\bar{b}$ Pert	$b\bar{b}$ Non-pert
A (GeV^2)	0.168	0.175	0.170	0.186
α_S	0.331	0.361	0.297	0.299
m_q (GeV)	1.41	1.49	5.14	6.33
μ (GeV)	2.32	1.07	4.79	3.61
f_V	0.00	0.18	0.00	0.09

2. RESULTS AND CONCLUSIONS

Our results[1] for the fit to the $c\bar{c}$ spectrum and the predicted $E1$ transition rates from the resulting wave functions are comparable to recent results for charmonium [6, 7], with the non-perturbative treatment yielding the best fit. The non-perturbative results[1] for the $b\bar{b}$ spectrum and decays are quite reasonable, though not as good as those from the perturbative treatment. In Table 2, we show the fit to the $b\bar{b}$ spectrum for the case of the perturbative treatment of V_L+V_S, and in Table 3, we show our predictions for the observed $E1$ transitions and the for the $E1$ decays associated with the $\Upsilon(1^3D_2)$.

Aside from the above threshold states in $c\bar{c}$, where mixing as well as continuum effects must be included to describe the $X(3872)$ and the $X(3943)$, both treatments of $c\bar{c}$ and $b\bar{b}$ yield very good overall fits. It is striking that for both systems the perturbative fits require the confining terms to be pure scalar, while the non-perturbative fits require a small amount of vector exchange.

[1] See: http://www.panic05.lanl.gov/sessions_by_date.php#sessions3

TABLE 2.

	Pert	Expt		Pert	Expt
$\eta_b(1S)$	9411.6	9300 ± 28	$\eta_b(3S)$	10339.5	
$\Upsilon(1S)$	9459.5	9460.3 ± 0.26	$\Upsilon(3S)$	10359.5	10355.2 ± 0.5
$1\chi_{b0}$	9862.5	9859.44 ± 0.52	$3\chi_{b0}$	10511.6	
$1\chi_{b1}$	9893.2	9892.78 ± 0.40	$3\chi_{b1}$	10534.5	
$1\chi_{b2}$	9914.0	9912.21 ± 0.17	$3\chi_{b2}$	10549.8	
$1h_b$	9902.1		$3h_b$	10540.9	
$\eta_b(2S)$	9996.5		1^3D_1	10149.8.	
$\Upsilon(2S)$	10020.9	10023.26 ± 0.31	1^3D_2	10157.6	10161.1 ± 1.7
$2\chi_{b0}$	10228.9	10232.5 ± 0.6	1^3D_3	10163.5	
$2\chi_{b1}$	10254.0	10255.46 ± 0.55	1^1D_2	10158.9	
$2\chi_{b2}$	10270.8	10268.65 ± 0.55			
$2h_b$	10261.1				

TABLE 3.

$\Gamma_\gamma(E1)$ (keV)	Pert	Expt	$\Gamma_\gamma(E1)$ (keV)	Pert	Expt
$\Upsilon(2S) \to \gamma\, 1\chi_{b0}$	1.12	1.16 ± 0.15	$\Upsilon(3S) \to \gamma\, 2\chi_{b0}$	1.64	1.30 ± 0.20
$\Upsilon(2S) \to \gamma\, 1\chi_{b1}$	1.79	2.11 ± 0.20	$\Upsilon(3S) \to \gamma\, 2\chi_{b1}$	2.61	2.78 ± 0.43
$\Upsilon(2S) \to \gamma\, 1\chi_{b2}$	1.76	2.19 ± 0.20	$\Upsilon(3S) \to \gamma\, 2\chi_{b2}$	2.59	2.89 ± 0.50
$2\chi_{b1} \to \gamma\,\Upsilon(1^3D_2)$	1.47		$2\chi_{b2} \to \gamma\,\Upsilon(1^3D_2)$	0.47	
$\Upsilon(1^3D_2) \to \gamma\, 1\chi_{b1}$	19.7		$\Upsilon(1^3D_2) \to \gamma\, 1\chi_{b2}$	5.16	

ACKNOWLEDGMENTS

This research was supported in part by the National Science Foundation under Grant PHY-0244789.

REFERENCES

1. J. Pumplin, W. W. Repko and A. Sato, Phys. Rev. Lett. **35**, 1538 (1975).
2. H. J. Schnitzer, Phys. Rev. Lett. **35**, 1540 (1975).
3. E. Eichten, K. Gottfried, T. Kinoshita, K. D. Lane, and T. M. Yan, Phys. Rev. D **17**, 3090 (1978).
4. S. N. Gupta, S. F. Radford and W. W. Repko, Phys. Rev. D **26**, 3305 (1982).
5. S. N. Gupta, J. M. Johnson, W. W. Repko and C. J. Suchyta III, Phys. Rev. D **49**, 1551 (1994).
6. E. Eichten, K. Lane and C. Quigg, Phys. Rev. D **69**, 094019 (2004); arXiv:hep-ph/0511179.
7. T. Barnes and S. Godfrey, Phys. Rev. D **72**, 054026 (2005) arXiv:hep-ph/0505002.
8. S. F. Radford and W. W. Repko, Int. J. Mod. Phys. A**20**, 3774 (2005) arXiv:hep-ph/0409290.