LIGHT-CONE REPRESENTATION of QUARK SCHWINGER-DYSON EQUATION
⇒ L-C Bethe-Salpeter

1. Review of Schwinger-Dyson formalism
   Bethe-Salpeter "
2. Obtaining a light-cone representation of the quark SDE
3. Models and constraints
4. Results with polynomial + instanton models. EXPLATORY

Otto Linsuain, CMU Ph.D. Dissertation (2002)

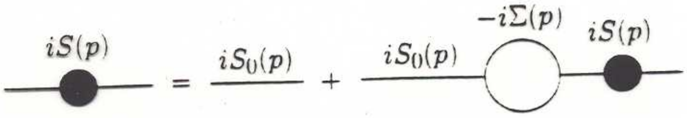
L. S. K & O. Linsuain, hep-ph/0110111

DRESSED QUARK PROPAGATOR

$S(p)$ = dressed quark propagator

$S_0(p)$ = bare propagator

$\Sigma(p)$ = self-energy



$$S^{-1}(p) = S_0^{-1}(p) - \Sigma(p)$$

$$S_0^{-1}(p)_{\mu^2} = Z_2(\mu^2)\,(i\not{p} + m_0)$$

For large $\mu^2 > 0$

$$S^{-1}_{\mu^2}(p) \xRightarrow[p^2=\mu^2]{} i\not{p} + m(\mu^2)$$

In general

$$S^{-1}(p) = A(p^2,\mu^2)\,i\not{p} + B(p^2,\mu^2)$$

$$M_{\mu^2}(p^2) \equiv B(p^2,\mu^2)/A(p^2,\mu^2)$$

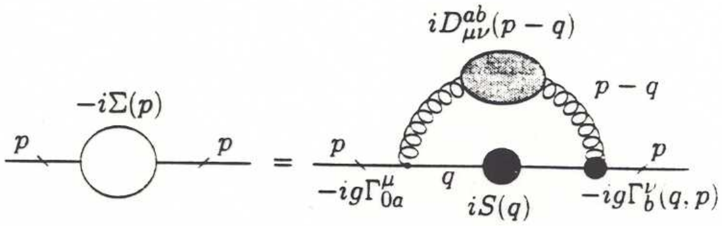
QUARK SCHWINGER-DYSON EQUATION

Dressed Gluon propagator $D^{ab}_{\mu\nu}(k) = \delta^{ab} D_{\mu\nu}(k)$

Dressed G-Q Vertex $\Gamma^{\nu}_{b}(q,p) = \Gamma^{\nu}(q,p)\lambda_b/2$

Bare G-Q vertex $\Gamma^{\mu}_{0a} = \gamma^{\mu}\lambda_a/2$

λ_a = SU(3) color matrices



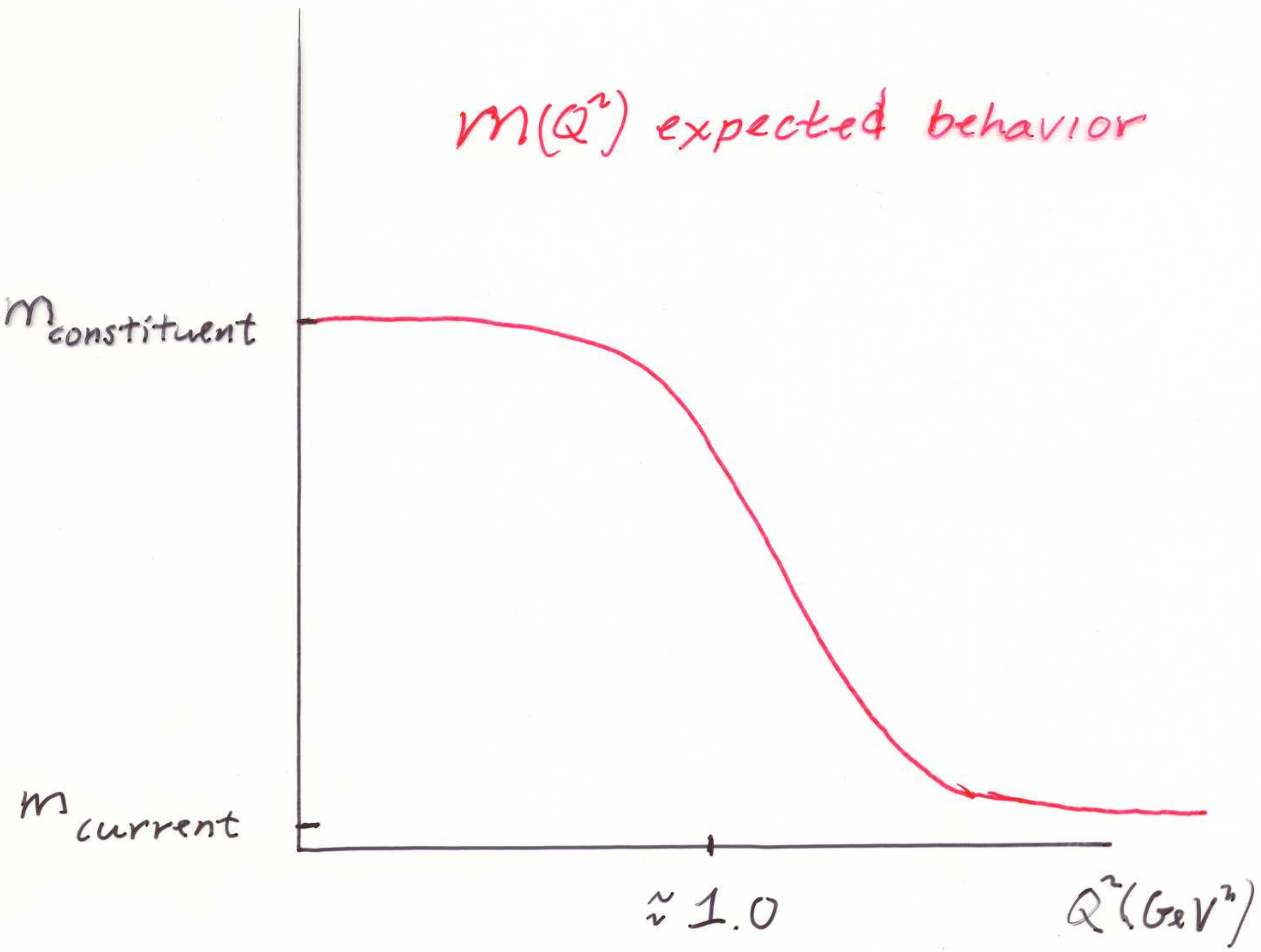
SDE in 4-D

$$\Sigma(p) = i g^2 \int \frac{d^4q}{(2\pi)^4} \Gamma^{\mu}_{0a} S(q) \Gamma^{\nu}_{b}(q,p) D^{ab}_{\mu\nu}(p-q)$$

Solutions: $A(p^2)$, $B(p^2)$

Need Γ^{ν}_{b}, $D^{ab}_{\mu\nu}$

RUNNING QUARK MASS



CONSTRAINTS on SDE SOLUTIONS

Quark Condensate

$$\langle 0| : \bar{q}(0)q(0) : |0\rangle = -\frac{3}{4\pi^2}\int_0^{+\infty} dS S \frac{B(S)}{SA^2(S)+B^2(S)}$$

$\approx (.225\ GeV)^3$

Mixed Condensate

$\langle 0| : \bar{q}(0)g\sigma \cdot G(0)q(0) : |0\rangle =$

$$\frac{9}{4\pi^2}\int_0^{+\infty} dS S \left\{ S\frac{B(S)(2-A(S))}{SA^2(S)+B^2(S)} + \frac{81B(S)[2SA(S)(A(S)-1)+B^2(S)]}{16(SA^2(S)+B^2(S))}\right\}$$

$\approx (.4 - .6\ GeV)^5$

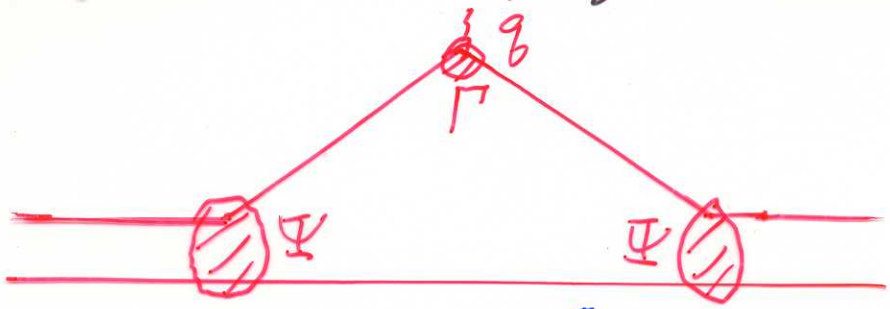
Pion Decay Constant

$$f_\pi^2 = \frac{3}{4\pi^2}\int_0^{+\infty} dS \frac{SM(S)}{A(S)[S+M^2(S)]^2}\left[M(S) - \frac{S}{2}\frac{dM}{dS}\right]$$

$\approx (.092\ GeV)^2$

LIGHT-CONE SDE
MOTIVATION: HADRONIC STRUCTURE

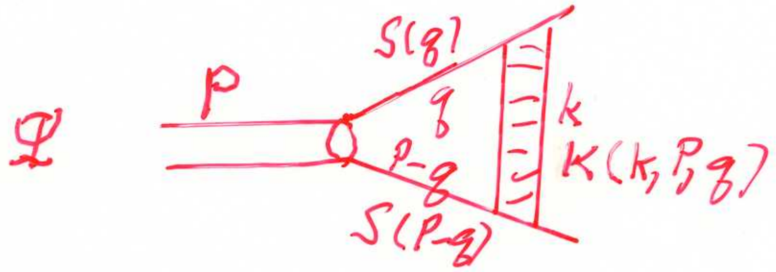
E.G. meson form factors, high momentum transfer



$$F(q) = \int dk\ \Psi^{\dagger}(k')\,\Gamma(k';k)\,\Psi(k)$$

Ψ = Bethe-Salpeter amplitude

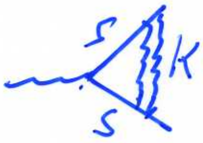
Jacob + Kisslinger P.R.L. 56 ('86); P.L. B243 (90)
Lapage + Brodsky P.R.D22 (1980)
Kisslinger, Choi, + Ji P.R. D63, (2001)
Bakker, Choi, + Ji PR D65 (2002)
P. Maris, C. Roberts, + P. Tandy (1997-2004)



To obtain l.c. $F(q)$ need:

l.c. $S(q)$ (from l.c. SDE)

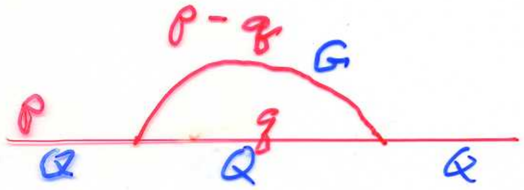
l.c. $K \Leftrightarrow D_{\mu\nu}(k)$ (l.c. SDE model)

l.c. $\Gamma \Rightarrow$ 

KEY: model $D_{\mu\nu}(k)$ on l.c.

$D_{\mu\nu}(k) \propto e^{-\beta^2 k^2}$ $\left\{\begin{array}{l}\text{easy integrations}\\ \text{no go singularities}\end{array}\right\}$

LIGHT-CONE REPRESENTATION of QUARK SDE



Q = quark
G = glue

Condider typical Feynman-like loop, d-dimensions

$$d(p) \equiv \int \frac{d^d q}{(2\pi)^d}\, F_Q(q^2+i\epsilon)\, F_G((p-q)^2+i\epsilon)$$

Light-cone variables

$$q^{\pm} = q^0 \pm q^3, \quad \vec{q}_\perp = (q^1, q^2)$$

Feynman Rules at ∞ momentum: (Chang-Ma, P.R. **180** (1969))
simplification of vacuum structure

$$\int_{-\infty}^{\infty} dq^+ \rightarrow \int_0^{p^+} dq^+$$

LC-SDE variables $\alpha = \frac{q^+}{p^+}$, $s' = q\cdot q$, $s = p\cdot p$, $q'_\perp = q_\perp - \alpha p_\perp$

$$d(p) \Rightarrow \int_{-\infty}^{\infty} \frac{ds'}{2\pi} \int_0^1 \frac{d\alpha}{4\pi} \int \frac{d^{d-2} q'_\perp}{(2\pi)^{d-2}}\, \frac{F_Q(s'+i\epsilon)}{\alpha}\, F_G\left(-\frac{q'^2_\perp - s\alpha(1-\alpha) + s'(1-\alpha) - i\epsilon}{\alpha}\right)$$

Because we are doing a self-consistent calculation to determine the singularity structure as well as eventually $A(p^2)$, $B(p^2)$ we cannot do the $\int dq^-$ as in standard perturbation theory or BSE (Lepage-Brodsky P.R. **22** (1980))

Minkowski Space Formulation

We work in Minkowski space

For consistency with light-cone field theory the α integral is in the range $0 \to 1$

Singularities ansatz: singularities of the F_Q & F_G functions must lie on or below the real axis

With our method we can extend the calculations to time-like regions

The instanton contributions are also treated in Minkowski space via analytic continuation

MODELS

I) Γ_a^μ One can use Slavnov-Taylor identities for transverse part of vertex + model longitudinal (Ball-Chiu)

All results today use rainbow ($\Gamma_a^\mu = \gamma^\mu \lambda_a/2$) in this <u>preliminary study</u>

II Models for gluon propagator

A] <u>Polynomial Model</u>

Landau gauge

$$D_{\mu\nu}(k) = \frac{-1}{k^2+i\epsilon}\left(g_{\mu\nu} - x\frac{k_\mu k_\nu}{k^2+i\epsilon}\right) D(k^2)$$

(Feynman-like gauge, $x=0$; Landau gauge $x=1$)

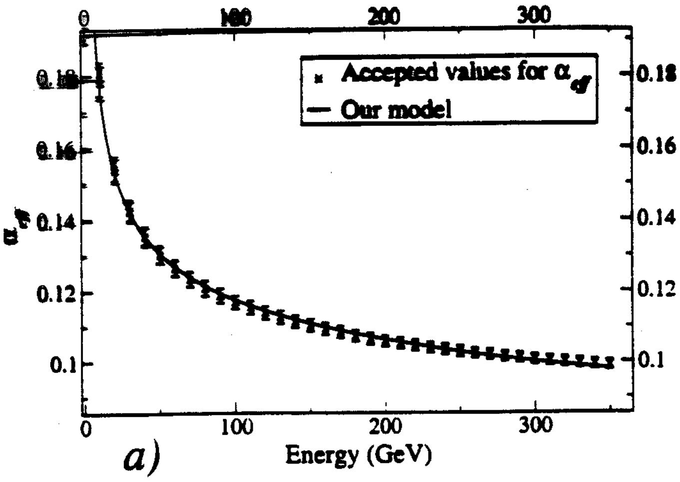
Motivated by $\frac{1}{k^4}$ confinement ('t Hooft, N.P. <u>75</u> (1974))

$$g^2 D(k^2) = 4\pi\left\{(-1)^{c_1}\lambda_1\left(\frac{s_0}{k^2+i\epsilon}\right)^{c_1} + (-1)^{c_2}\lambda_2\left(\frac{s_0}{k^2+i\epsilon}\right)^{c_2}\right\}$$

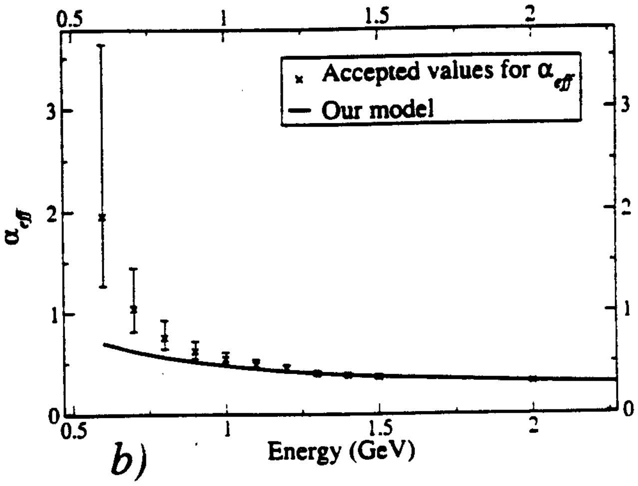
$$g^2 D(k^2) = 4\pi\,\alpha_{eff}(k^2)$$

With $s_0 = 1$ GeV choose $c_1, \lambda_1, c_2, \lambda_2$ to fit PDG α_{eff} for $1.3 < E < 350$ GeV

Fit to $\alpha_{eff}(k^2)$



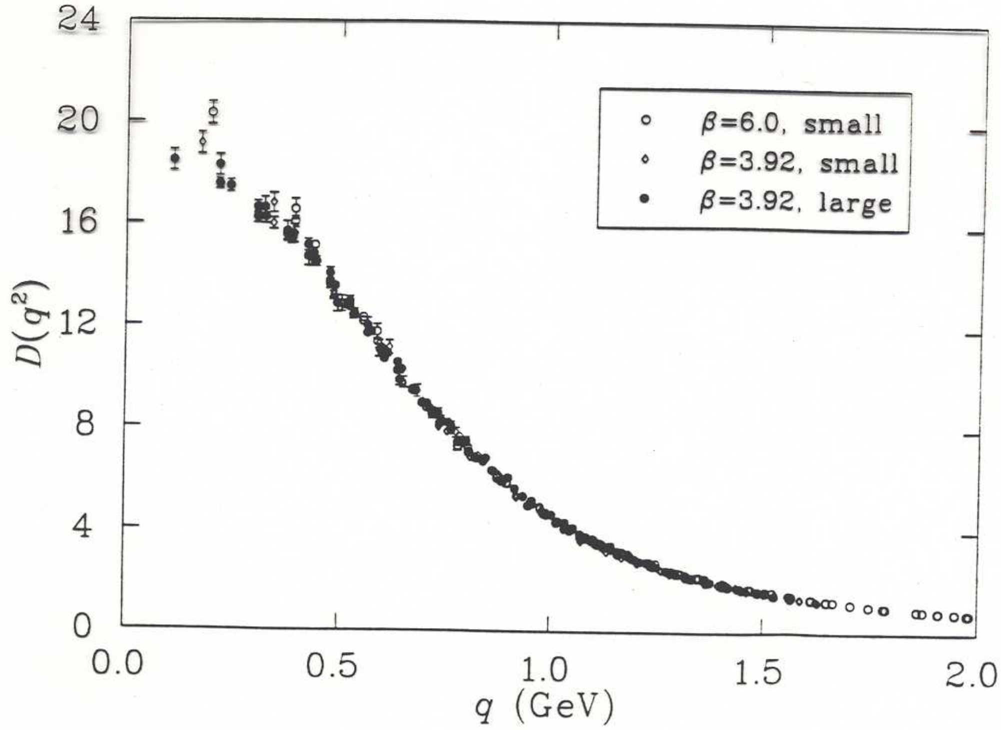
a)



b)

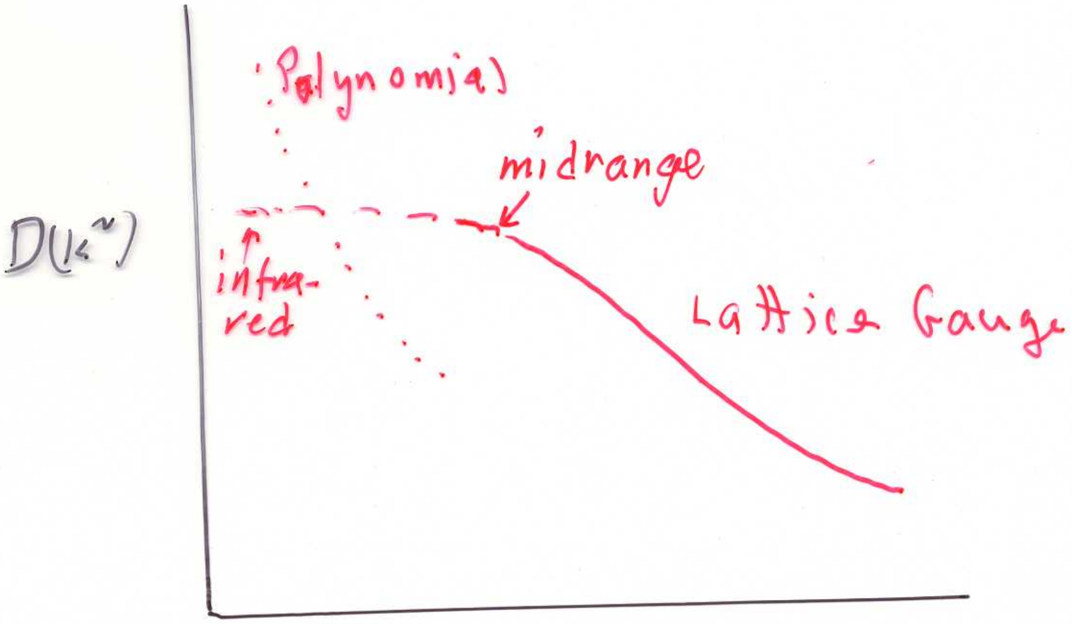
LATTICE CALCULATION of GLUON PROPAGATOR

Bonnet, Bowman, Leinweber + Williams
P.R. D62, 051501 (2000)



Infrared reduced, mid-range enhanced

LATTICE GAUGE $D(k^2)$ & POLYNOMIAL MODEL



Polynomial model: too strongly infra-red enhanced

Instantons: Provide midrange nonperturbative QCD effects
Do not confine (need infra-red)

Our Models: Polynomial + Instanton

B INSTANTON GLUON PROPAGATOR

$$\mathcal{L}^{QCD} = \mathcal{L}^{Glue} + \mathcal{L}^{quark-glue}$$

$$\mathcal{L}^{Glue} = -\frac{1}{4} G^a_{\mu\nu} G^{\mu\nu a} = \mathcal{L}^{(Inst)} + \mathcal{L}^{(quantum)}$$

$$A^a_\mu(x)^{inst} = 2\eta^a_{\mu\nu} \frac{x_\nu}{(x^2+\rho^2)}$$ $\eta^a_{\mu\nu}$ = numerical

Belavin et.al. P.L. B59 (1975)

$$\mathcal{L}^{(inst)} = -\frac{1}{4} \frac{192\rho^4}{(x^2+\rho^2)^4}$$

Liquid instanton model (Shuryak et.al.) $\rho = 1\frac{2}{3}\,GeV^{-1}$

Quark propagator in $I + \bar{I}$ medium

Pobylitsa, P.L. 226 (1989)

$$S_I^{-1}(p) = \not{p} - B_I(p^2) \qquad A_I = 1$$

$$B_I(P) = K\, P^2 f^2\!\left(\frac{\rho P}{2}\right)$$

$$f(z) = \frac{2}{z} - (3I_0(z) + I_2(z))\,K_1(z)$$

$$K \simeq 0.29\,GeV^{-1}$$

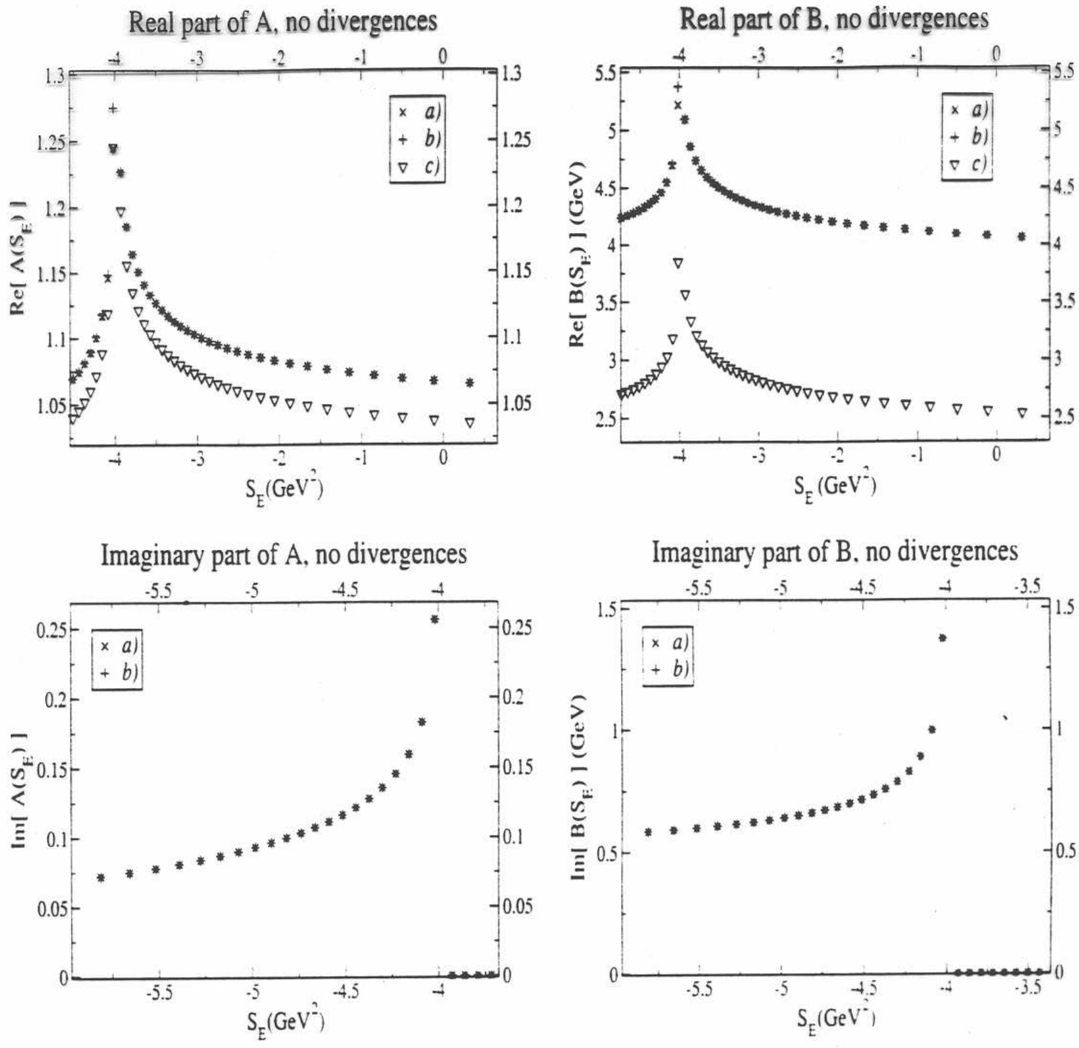
Note, the Instantons are found in Euclidean space.

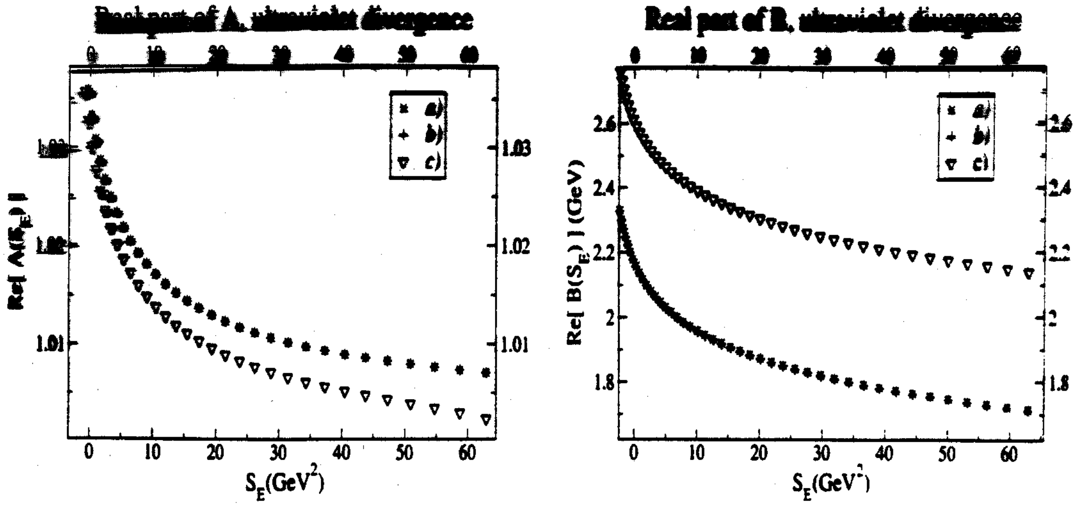
Numerical Accuracy

One iteration

× analytical

+ numerical




Real part of B, ultraviolet divergence
0 10 20 30 40 50 60
a)
b)
c)
1.03
1.02
1.01
2.6
2.4
2.2
2
1.8
Re[B(S_E)] (GeV)
S_E(GeV^2)

RESULTS WITH VARIOUS MODELS

I Polynomial Model

I $\boxed{S^{-1}(p) = S_0^{-1}(p) - \Sigma_p(S)}$

$\Sigma_p(S) \equiv$ self energy calculated [self consistently] with S

Two sets

$1: \begin{cases} \lambda_1 = .222 & c_1 = 0.07 \\ \lambda_2 = .25 & c_2 = 0.6 \end{cases}$

$2: \begin{cases} \lambda_1 = .222 & c_1 = 0.07 \\ \lambda_2 = 1.5 & c_2 = 0.08 \end{cases}$

II Bare quark ⇒ quark in (I, I) medium

II $\boxed{S^{-1}(p) = S_I^{-1}(p) - \Sigma_p(S)}$

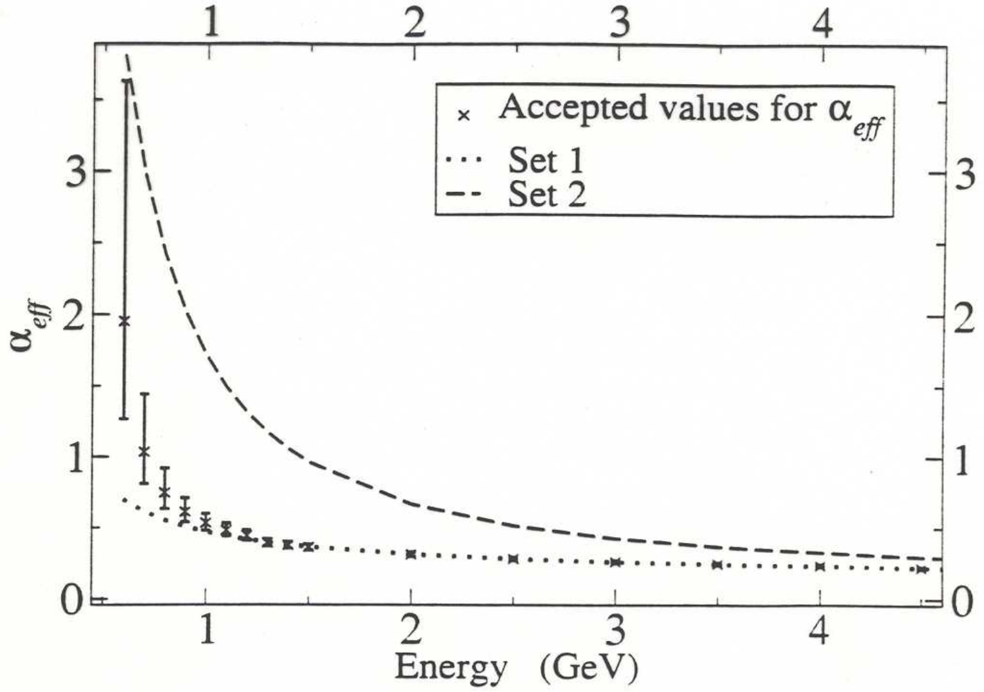
III Add S_I self energy in model I to I

III $\boxed{S^{-1}(p) = S_0^{-1}(p) - \Sigma_p(S) - \Sigma_p(S_I)}$

IV $\boxed{S^{-1}(p) = S_I^{-1}(p) - \Sigma_p(S) - \Sigma_p(S_I)}$

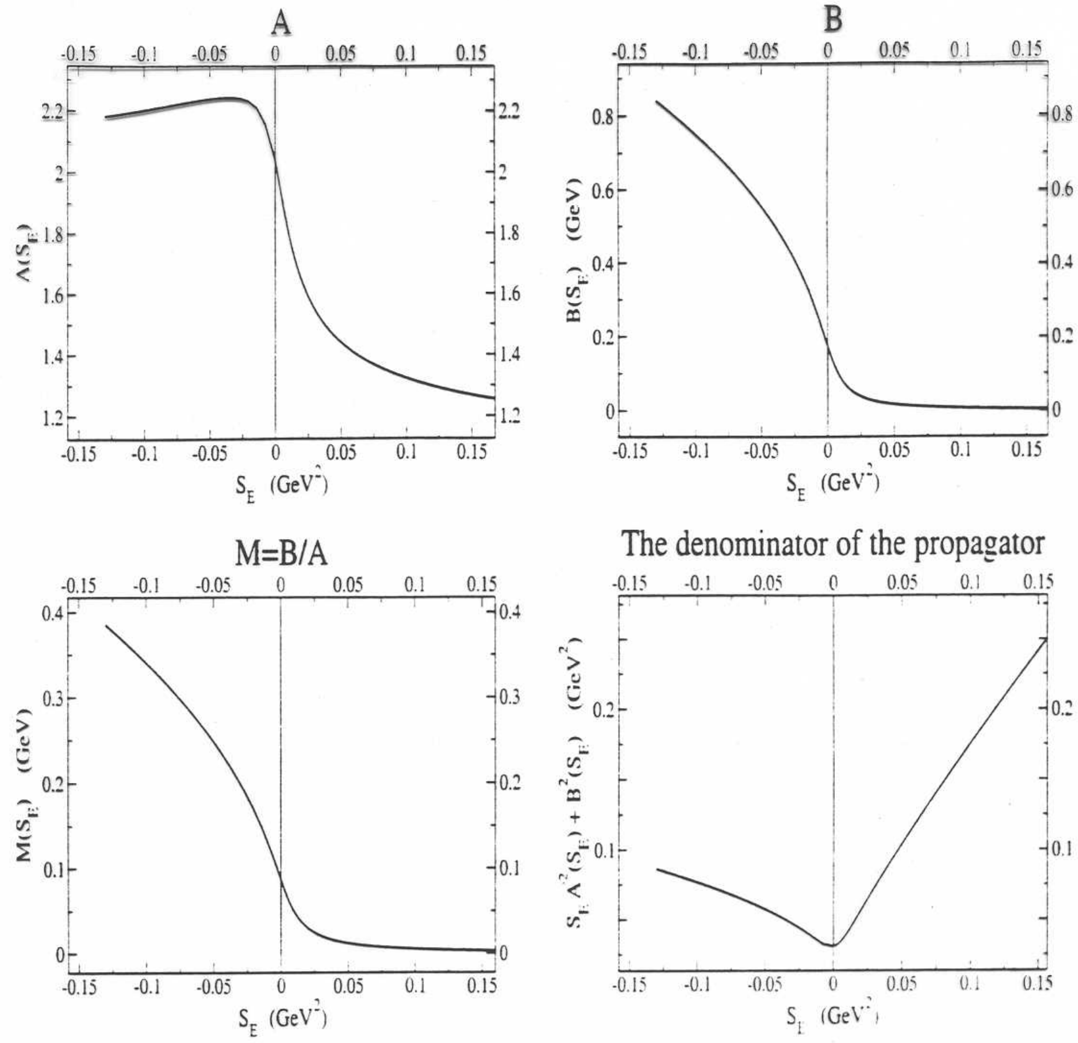
Polynomial Model

Fits to $\alpha_{eff}(k)$ with two sets

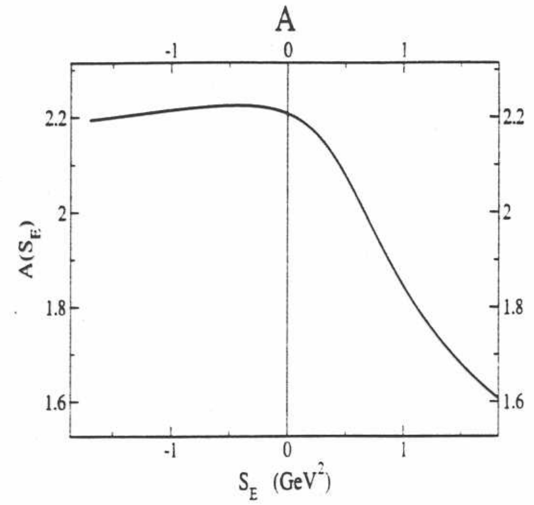


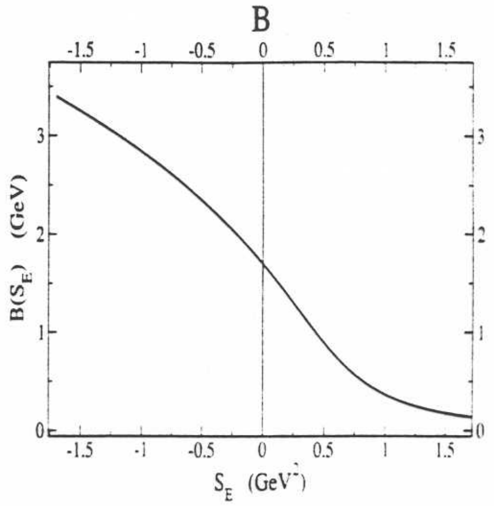
Set 2 not good, but improves fits to constraints (discussed below)

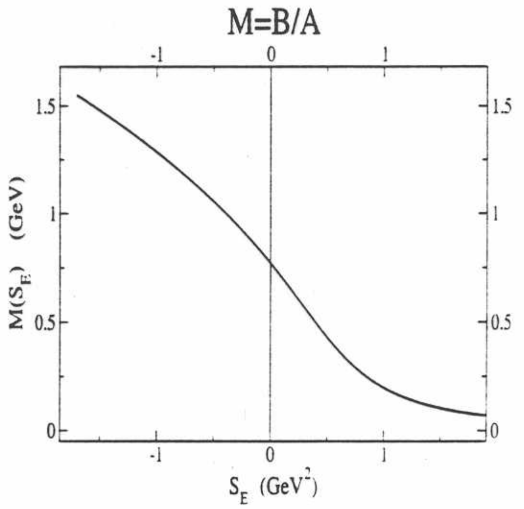
Polynomial Model/No Instantons
Set 1

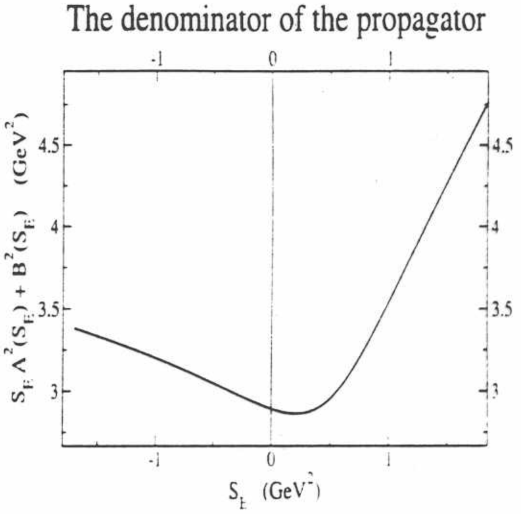


Polynomial Model/No Instantons
Set 2









RESULTS, FOUR MODELS

	Vertex: $\Gamma^\nu = \gamma^\nu$ Rainbow approximation								
	model	$(-\langle : \bar{q}q : \rangle)^{1/3}$ (MeV)		$(-\langle : \bar{q}g\sigma \cdot Gq : \rangle)^{1/5}$ (MeV)		f_π (MeV)		$M(0)$ (MeV)	
		Set 1	Set 2	Set 1	Set 2	Set 1	Set 2	Set 1	Set 2
I	Eq. (27)	58	288.5	313.5	1111.5	13	89	86.5	770.5
II	Eq. (28)	221	227.5	558	906.5	91.5	117.5	534	1084.5
III	Eq. (29)	171.5	153.5	509	975.5	58.5	54.5	308	752.5
IV	Eq. (30)	219	159.5	624	1001	91	60	607	804
	For comparison								
	Eqs. (15)	216.7		456.5		86		417.6	
"Exp."	Other calc.	200-250		400-600		92		~ 300	
		Sum rules, lattice QCD				Exper.		NRQM[a]	

CONCLUSIONS

We have shown how to obtain a light-cone form for the quark SDE

Our models include enhanced infrared behavior for confinement + instanton effects for mid-range QCD interaction

With polynomial one can work in Minkowski space. With instantons an analytic continuation to Minkowski space is needed

For good fits to NPQCD constraints (condensates, f_π) instanton + polynomial works o.k.

This is an exploratory work. It can provide a framework for more complete solutions of the quark propagator