Baryon Resonances from a FLIC Fermion Action in Lattice QCD

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Outline

- Introduction to lattice QCD
- What are FLIC fermions?
  - Clover Fermions - mean-field improved
- non-perturbative improved
  - APE Smearing and Improvement of Lattice Operators

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  - Near continuum results at finite lattice spacing.
- Access to Light Quark Masses
  - $m_\pi / m_\rho = 0.35$

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- Baryon Resonance Phenomenology
  - $N^* 1/2^+ \,,\; N^* 1/2^- \,,\; N^* 3/2^+ \,,\; N^* 3/2^- \,,\; \Delta^* 3/2^- \,,\; \Delta^* 1/2^\pm .$

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- Future Outlook

Introduction to lattice QCD

- Complete solution of QCD $\equiv$ knowing all possible Minkowski space Green's functions of the theory.
- Implies for every possible combination of quark and gluon operators, $O[\hat{A},\hat{\bar{q}},\hat{q}]$, we need to know

$$\langle\Omega|\hat{T}\left(O[\hat{A},\hat{\bar{q}},\hat{q}]\right)|\Omega\rangle = \frac{\int \mathcal{D}A\mathcal{D}\bar{q}\mathcal{D}q\, O[A,\bar{q},q]\exp(iS[A,\bar{q},q])}{\int \mathcal{D}A\mathcal{D}\bar{q}\mathcal{D}q\, \exp(iS[A,\bar{q},q])}$$

$$= \frac{\int \mathcal{D}A \,\det\left[S_F^{-1}[A]\right]\, O[A,S_F[A]]\exp(iS[A])}{\int \mathcal{D}A\,\det\left[S_F^{-1}[A]\right]\,\exp(iS[A])},$$

- Note that $S[A]$ is the pure gluon (i.e., pure gauge) action.
- $|\Omega\rangle \equiv$ nonperturbative vacuum, $\hat{T} \equiv$ time-ordering operator, $S_F([A];x,y) \equiv$ tree-level quark propagator in gluon field, A.
- $O[A,S_F[A]] \equiv \{O[A,\bar{q},q]$ with every possible pairwise contraction of $\bar{q}$ and q replaced by the propagator $S_F([A];x,y)\}$

Introduction to lattice QCD (contd)

- It is numerically convenient to work in Euclidean space, where all quantities are now Euclidean. So we need to know

$$\langle\Omega|\hat{T}\left(O[\hat{A},\hat{\bar{q}},\hat{q}]\right)|\Omega\rangle \equiv \frac{\int \mathcal{D}A\mathcal{D}\bar{q}\mathcal{D}q\, O[A,\bar{q},q]\exp(-S[A,\bar{q},q])}{\int \mathcal{D}A\mathcal{D}\bar{q}\mathcal{D}q\, \exp(-S[A,\bar{q},q])}$$

$$= \frac{\int \mathcal{D}A\, O[A,S_F[A]]\det\left[S_F^{-1}[A]\right]\exp(-S[A])}{\int \mathcal{D}A\, \det\left[S_F^{-1}[A]\right]\exp(-S[A])}$$

- There is one factor of $\det[S_F^f[A]]$ for each quark flavor f, i.e., we use the notation $\det[S_F^{-1}[A]] \equiv \Pi_f \det[S_F^f[A]]$.
- Can study many observables from Euclidean space.
- We will only sample gauge inequivalent A's and hence for observables (i.e., for color singlet $O[\hat{A},\hat{\bar{q}},\hat{q}]$) $\Rightarrow$ won't have to bother with gauge fixing!

Introduction to lattice QCD (contd)

- The lattice approximates infinite Euclidean space by a four-dimensional discrete space-time lattice, where
  - $a \equiv$ lattice spacing, (typically $0.1 \sim 0.2$ fm).
  - N_s and N_t are number of lattice sites in space and time directions respectively.
  - $L_s = N_s a$ and $L_t = N_t a$ are physical length of lattice in space and time directions respectively.
  - $V = {L_s}^3 \times L_t \equiv$ physical lattice volume; ${N_s}^3 \times N_t \equiv$ lattice volume in lattice units.
- Introduce links, $U_\mu(x) \equiv U(x, x + a\mu) \in SU(3)$, between sites in the Cartesian directions $\mu = 1, \cdots, 4$. Links replace the gluon fields $A_\mu(x) \equiv \sum_{a=1}^{8} A_\mu^a(x)(\lambda^a/2) \in SU(3)$.
- Links are parallel transport operators $U_\mu(x) = \hat{P} \exp\left(ig_s \int_x^{x+a\mu} dx' \cdot A(x')\right)$, where $\hat{P} \equiv$ path ordering.

Introduction to lattice QCD (contd)

- We can express the gauge field in terms of finite differences of links, i.e., we can always express $A_\mu(x)$ as $A_\mu([U],x)$.
- We can generate an ensemble of gauge field configurations, $\{U_1, \cdots, U_{N_{\rm cf}}\}$ weighted with the probability distribution $P[U] \propto \det[S_F^{-1}[U]] \exp(-S[U]) \equiv \Pi_f \det[S_F^f[U]] \exp(-S[U])$.
  - Will never have two gauge equivalent configurations in a finite ensemble $\Rightarrow$ no gauge fixing needed.
  - Since we need $1 \geq P[U] \geq 0$ and since $\det[S_F^f[A]]$ is real, then we want to simulate with degenarate mass flavor pairs, (i.e., so that $\det[S_F^{-1}[U]] \equiv \Pi_f \det[S_F^f[A]] \geq 0$).
  - Techniques exist for unpaired flavors, but are more difficult.
  - We frequently approximate $P[U] \propto \exp(-S[U])$, which omits the determinant and is equivalent to omitting all quark loops $\Rightarrow$ this is called the quenched approximation.

Introduction to lattice QCD (contd)

- Hence we can now evaluate the Euclidean Green's function for any color-singlet $O[\cdots]$ by simply taking its ensemble average

$$\langle\Omega|\hat{T}\left(O[\hat{A},\hat{\bar{q}},\hat{q}]\right)|\Omega\rangle \equiv \langle O[U,S_F[U]]\rangle$$

$$= \frac{\int \mathcal{D}U\, O[U,S_F[U]]\det\left[S_F^{-1}[U]\right]\exp(-S[U])}{\int \mathcal{D}U\, \det\left[S_F^{-1}[U]\right]\exp(-S[U])}$$

$$= \lim_{V\to\infty}\lim_{a\to 0}\lim_{N_{\rm cf}\to\infty}\frac{\sum_{i=1}^{N_{\rm cf}} O[U_i,S_F[U_i]]}{\sum_{i=1}^{N_{\rm cf}}}$$

$$= \lim_{V\to\infty}\lim_{a\to 0}\lim_{N_{\rm cf}\to\infty}\frac{1}{N_{\rm cf}}\sum_{i=1}^{N_{\rm cf}} O[U_i,S_F[U_i]]$$

Observables from Euclidean Space

- We move from Minkowski space $\rightarrow$ Euclidean space by the analytic continuation: $t \rightarrow -it_E$ or in a different notation $x^0 \rightarrow -ix_4$.
- Thus the Minkowski-space evolution operator, becomes the Euclidean-space version: $\exp(-i\hat{H}t) \rightarrow \exp(-\hat{H}t_E)$.
- Note that replacing t_E with $\beta \equiv 1/kT$ and taking the trace gives the partition function of statistical mechanics:

$$Z(\beta) \equiv \mathrm{tr}[\exp(-\beta\hat{H})] = \sum_n \exp(-\beta E_n).$$

- Consider ordinary Quantum Mechanics in the presence of some conserved charge operator $\hat{Q}$. Since $[\hat{H}, \hat{Q}] = 0$ we have:
  - $\hat{H}|E_n^q\rangle = E_n|E_n^q\rangle$ and $\hat{Q}|E_n^q\rangle = q|E_n^q\rangle$, where E_n and q are the energy and charge e-values respectively.
  - $\Rightarrow$ Hilbert space is divided up into charge sectors labelled by q, where *any* state $|\chi^q\rangle$ in the q charge sector can be written as: $|\chi^q\rangle = \sum_n c_n|E_n^q\rangle$ for some c_n.

Observables from Euclidean Space (contd)

- Let $|\Omega\rangle$ be the ground state (i.e., vacuum) of the system. Then
$$\hat{H}|\Omega\rangle = \hat{Q}|\Omega\rangle = 0$$
- Define the Schrödinger picture operators $\hat{\bar{\chi}}^q$ and $\hat{\chi}^q$ such that
$$\langle\chi^q| = \langle\Omega|\hat{\chi}^q \qquad \text{and} \qquad |\chi^q\rangle = \hat{\bar{\chi}}^q|\Omega\rangle.$$
- In Euclidean space the Heisenberg picture operators are:
  - $\hat{\bar{\chi}}^q(t_E) \equiv \exp(+\hat{H}t_E)\,\hat{\bar{\chi}}^q\,\exp(-\hat{H}t_E)$
  - $\hat{\chi}^q(t_E) \equiv \exp(+\hat{H}t_E)\,\hat{\chi}^q\,\exp(-\hat{H}t_E)$.
- Then we can define the correlation function:
$$\begin{aligned} G(t_E) &\equiv \langle\Omega|\hat{\chi}^q(t_E)\hat{\bar{\chi}}^q(0)|\Omega\rangle = \langle\chi^q|\exp(-\hat{H}t_E)|\chi^q\rangle \\ &= \sum_{n=0} |c_n|^2 \exp(-E_n^q t_E) \end{aligned}$$
- For large t_E can extract first few energies in the q charge sector, e.g., $E_0^q = \lim_{t_E\to\infty}(1/t_E)\ln G(t_E)$, *etc.*

Setting The Scale

- Use the Static Quark Potential

$$V(\mathbf{r}) = V_0 + \sigma r - e\left[\frac{1}{\mathbf{r}}\right] + l\left(\left[\frac{1}{\mathbf{r}}\right] - \frac{1}{r}\right)$$

where $\sqrt{\sigma} = 440$MeV and $\left[\frac{1}{\mathbf{r}}\right]$ denotes the tree-level lattice Coulomb term

$$\left[\frac{1}{\mathbf{r}}\right] = 4\pi \int \frac{d^3\mathbf{k}}{(2\pi)^3} \cos(\mathbf{k}\cdot\mathbf{r}) D_{00}(0,\mathbf{k}),$$

and $D_{00}(k)$ is the time-time component of the gluon propagator.

- In the continuum limit,

$$\left[\frac{1}{\mathbf{r}}\right] \to \frac{1}{r}$$

Orion supercomputer

Orion supercomputer

- The calculations reported here were carried out on the Orion supercomputer. Orion consists of
  - 40 Enterprise E420R Sun nodes
  - Each node has 4 Ultrasparc II 450 MHz processors
  - Each of the 160 processors has 1 GByte of RAM and 4 MBytes of cache
  - All 40 nodes are interconnected by both Myrinet and fast ethernet
  - Each node has a peak speed of 3.6 Gflops
- Orion has a total peak theoretical speed of 144 Gflops and has 160 GBytes of RAM and 640 MBytes of cache.
- Orion has a measured performance of 110 Gflops with the Linpack benchmark
- Here is a photo of Orion.

Naive Lattice Fermion Action

- The continuum Dirac operator,

$$\not{D} = \gamma^{\mu}(\partial_{\mu} + i\, g\, A_{\mu})\,,$$

  is discretized by:

  - Replacing the derivative with a discrete difference, and
  - Including gauge links which
    - Encode the gluon field, A_{μ}, and
    - Maintain gauge invariance.

$$\bar{\psi}\,\not{D}\psi = \frac{1}{2a}\bar{\psi}(x)\sum_{\mu}\gamma_{\mu}\Big[U_{\mu}(x)\psi(x+\hat{\mu}) - U_{\mu}^{\dagger}(x-\hat{\mu})\psi(x-\hat{\mu})\Big].$$

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- The continuum Dirac action is recovered in the limit $a \to 0$ by Taylor expanding the U_{μ} and $\psi(a+\hat{\mu})$ in powers of the lattice spacing a.

The Naive Action (2)

- Hence we arrive at the simplest ("naive") lattice fermion action,

$$\begin{aligned} S_N &= m_q \sum_x \bar{\psi}(x)\psi(x) \\ &+ \frac{1}{2a}\sum_x \bar{\psi}(x)\gamma_\mu \Big[U_\mu(x)\psi(x+\hat{\mu}) - U^\dagger_\mu(x-\hat{\mu})\psi(x-\hat{\mu}) \Big] . \end{aligned}$$

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- While preserving chiral symmetry, encounters the fermion - doubling problem (i.e., it gives rise to $2^d = 16$ flavours rather than one).

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- While preserving chiral symmetry, encounters the fermion - doubling problem (i.e., it gives rise to $2^d = 16$ flavours rather than one).
- This doubling problem is demonstrated by the inverse of the free field propagator (obtained by taking the fourier transform of the action with all $U_\mu = 1$).

$$S^{-1}(p) = m_q + \frac{i}{a} \sum_\mu \gamma_\mu \sin p_\mu a$$

which has 16 zeros within the Brillouin cell in the limit $m_q \to 0$. eg,

$$\begin{aligned} p_\mu &= (0,0,0,0) \\ &= (\pi/a,0,0,0) \\ &= (\pi/a,\pi/a,0,0),\ \text{etc.} \end{aligned}$$

The Wilson Action

- Wilson introduced an irrelevant (energy) dimension-five operator (the so-called Wilson term) to fix this problem,

$$M_W = m_0 + \sum_{\mu} \left(\gamma_\mu \nabla_\mu - \frac{1}{2} r a \Delta_\mu \right) ,$$

where

$$\nabla_\mu \psi(x) = \frac{1}{2a} [U_\mu(x)\psi(x+\hat{\mu}) - U_\mu^\dagger(x-\hat{\mu})\psi(x-\hat{\mu})]$$

and

$$\Delta_\mu \psi(x) = \frac{1}{a^2} [U_\mu(x)\psi(x+\hat{\mu}) + U_\mu^\dagger(x-\hat{\mu})\psi(x-\hat{\mu}) - 2\psi(x)].$$

The Wilson Action

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- The Wilson action is (in terms of $U_\mu(x)$),

$$
\begin{aligned}
S_W &= \left(m_q + \frac{4r}{a}\right) \sum_x \bar{\psi}(x)\psi(x) \\
&+ \frac{1}{2a} \sum_{x,\mu} \bar{\psi}(x) \Big[(\gamma_\mu - r) U_\mu(x) \psi(x+\hat{\mu}) \\
&- (\gamma_\mu + r) U_\mu^\dagger(x-\hat{\mu}) \psi(x-\hat{\mu}) \Big]
\end{aligned}
$$

which explicitly breaks chiral symmetry at $\mathcal{O}(a)$.

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- This action has large $\mathcal{O}(a)$ errors $\rightarrow$ bad scaling.
- The scaling properties of this Wilson action at finite a can be improved by introducing any number of irrelevant operators of increasing dimension which vanish in the continuum limit.
- In this manner, one can improve fermion actions at finite a by combining operators to eliminate $\mathcal{O}(a)$ and perhaps $\mathcal{O}(a^2)$ errors etc.

The Clover Action

- The Clover action introduces an additional irrelevant dimension-five operator to remove $\mathcal{O}(a)$ errors.

$$S_{SW} = S_W - \frac{iaC_{SW}r}{4}\bar{\psi}(x)\sigma_{\mu\nu}F_{\mu\nu}\psi(x).$$

where C_{sw} is the clover coefficient which can be tuned to remove all $\mathcal{O}(a)$ artifacts.

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$$C_{sw} = 1 \text{ at tree-level.}$$

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$$u_0 = \left(\frac{1}{3}\mathcal{R}e\,\mathrm{tr}\langle U_{\mathrm{sq}}\rangle\right)^{1/4}$$

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- Insufficient to remove $\mathcal{O}(a)$ errors to all orders in g.

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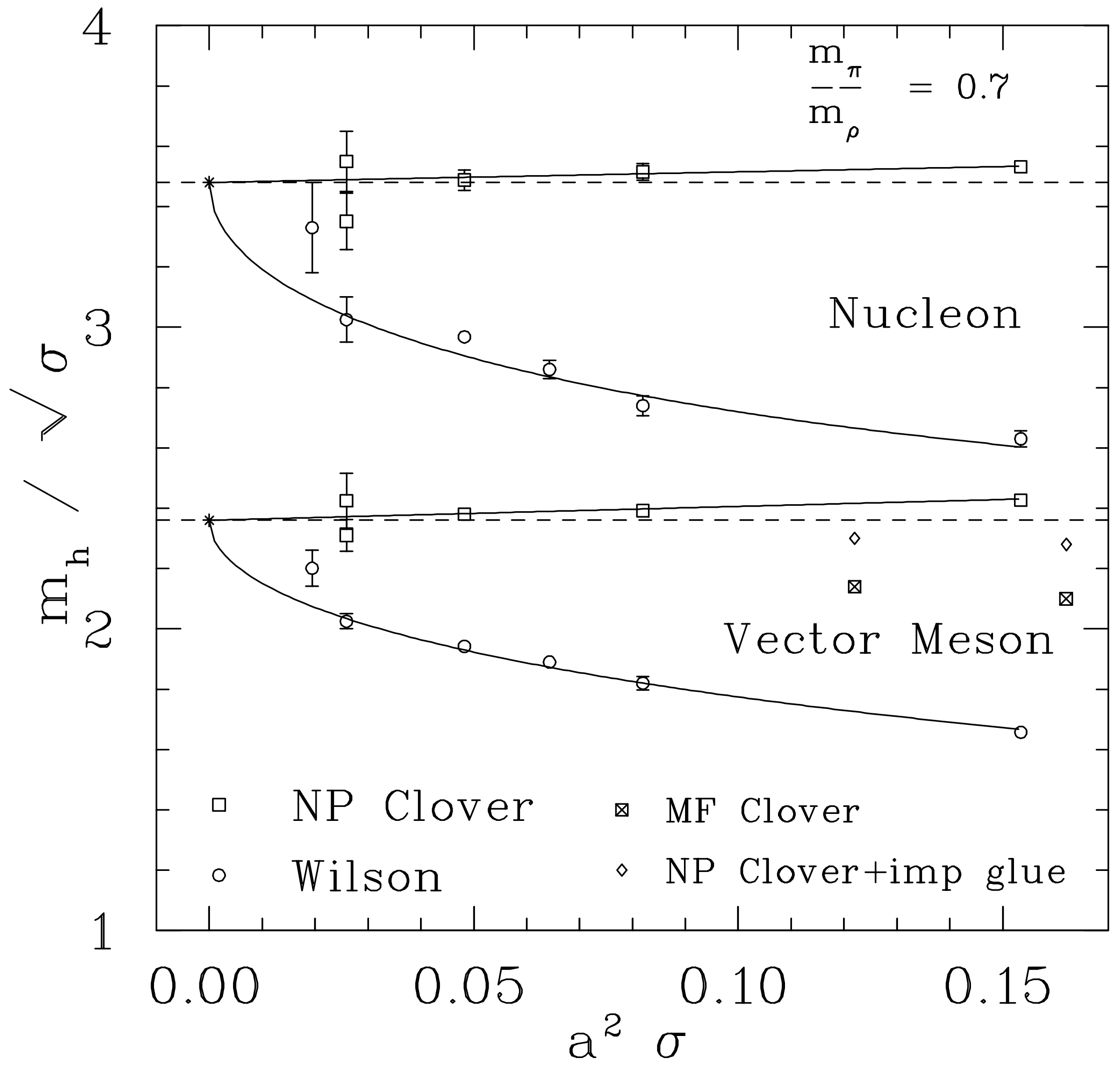
- The difficulty lies in determining the precise renormalization of C_{sw} in the interacting theory.
- Non-perturbative $\mathcal{O}(a)$ improvement (ALPHA Collaboration) - tune C_{sw} to all powers in g^2.
- The NP-Improved Clover action displays excellent scaling.

Clover Scaling Edwards, Heller, Klassen, PRL 80:3448-3451, 1998



The Difficulties with NP Clover

- The Clover action is responsible for revealing the exceptional configuration problem.
- The quark propagator encounters singular behaviour
  - as the quark mass becomes light,
  - as the lattice spacing becomes large.
- Chiral symmetry breaking in the action shifts continuum zero modes into the negative mass region.
- Accessing the light-quark mass regime is
  - Computationally intensive.

Difficulties with NP Clover

- The single plaquette-based $F_{\mu\nu}$ has large $\mathcal{O}(a^2)$ errors.
- Constructing the topological charge via

$$Q = \sum_x q(x) = \sum_x \frac{g^2}{32\pi^2} \epsilon_{\mu\nu\rho\sigma} \, \mathrm{Tr} \, \left(F_{\mu\nu}(x) \, F_{\rho\sigma}(x) \right)$$

- Implementing on smooth (cooled) configurations
- Reveals $\sim 10\%$ error in topological charge [F.D.R.Bonnet *et.al*, Phys.Rev.D62:094509,2000]

Fat-Link Fermion Actions

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- This process is called APE smearing.

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- The solution to these problems may be to work with two sets of links in the fermion action.

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- Drawbacks
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  - One loses short-distance quark interactions.
- Relevant dimension-four operators are constructed with untouched Monte-Carlo generated links.
- Irrelevant operators are constructed with smeared fat links.

Fat-Link Fermion Action (3)

- Mean-field improved Fat-Link Irrelevant Wilson action

$$
\begin{aligned}
S_W^{FL} &= \left(m_q + \frac{4r}{a}\right) \sum_x \bar{\psi}(x)\psi(x) \\
&+ \frac{1}{2a} \sum_{x,\mu} \bar{\psi}(x) \left[\gamma_\mu \left(\frac{U_\mu(x)}{u_0} \psi(x+\hat{\mu}) - \frac{U_\mu^\dagger(x-\hat{\mu})}{u_0} \psi(x-\hat{\mu}) \right) \right. \\
&- \left. r \left(\frac{U_\mu^{FL}(x)}{u_0^{FL}} \psi(x+\hat{\mu}) + \frac{U_\mu^{FL\dagger}(x-\hat{\mu})}{u_0^{FL}} \psi(x-\hat{\mu}) \right) \right]
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\end{aligned}
$$

- Mean-field improved Fat-Link Irrelevant Clover (FLIC) action

$$
S_{SW}^{FL} = S_W^{FL} - \frac{iaC_{SW}r}{4(u_0^{FL})^4} \bar{\psi}(x) \sigma_{\mu\nu} F_{\mu\nu} \psi(x).
$$

where $F_{\mu\nu}$ is constructed using fat-links.

FLIC Fermion Action

- Fat-link mean-field improvement parameter $u_0^{FL} \to 1$.

n	u_0^{FL}	$(u_0^{FL})^4$
0	0.889	0.624
4	0.997	0.986
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- Perturbative corrections are negligible after four sweeps.
- Mean-field improved estimate of coefficients is sufficient.
- Highly improved actions with many irrelevant operators (eg. D234) can be handled with confidence.
- Highly improved definitions of $F_{\mu\nu}$ involving terms up to u_0^8 may be used.

Lattice Simulations

- Calculations were performed using a mean-field improved, plaquette + rectangle, gauge action on a $16^3 \times 32$ lattice at $\beta = 4.60\,(\beta = 6/g^2)$, with lattice spacing $a = 0.122(1)$fm.
- Fixed boundary condition in time direction, ie.

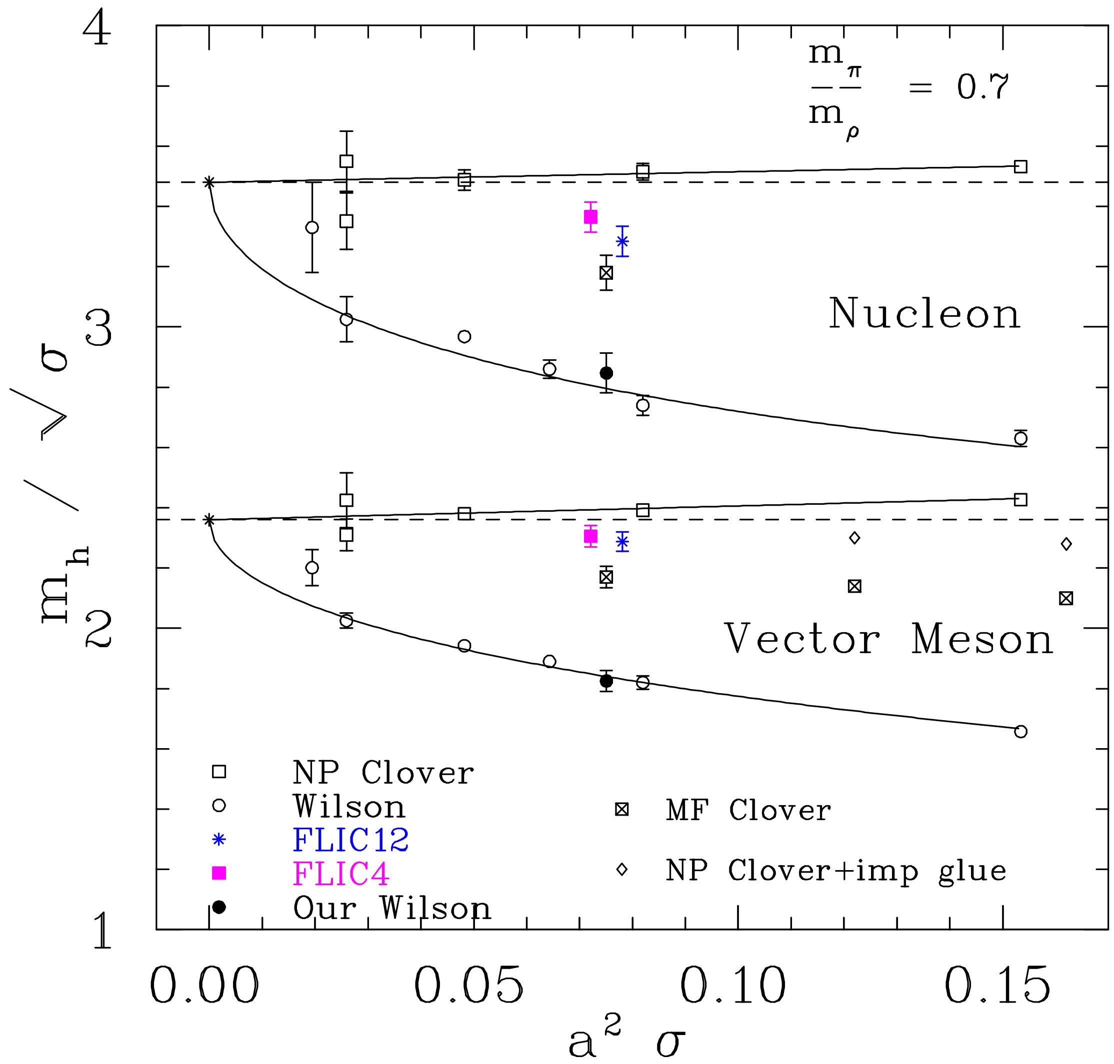
$$U_t(\vec{x}, nt) = 0 \qquad \forall \vec{x}$$

- The source was created at a space-time location of $(x, y, z, t) = (1, 1, 1, 3)$.
- Gauge-invariant gaussian smearing was applied at the source to increase the overlap of the interpolating operators with the ground states.

The Lattices

β	a(fm)	$L^3 \times T$	Length(fm)
4.38	0.165	$12^3 \times 24$	1.980
4.60	0.122	$12^3 \times 24$	1.464
4.60	0.122	$16^3 \times 32$	1.952
4.80	0.093	$16^3 \times 32$	1.488

FLIC Scaling



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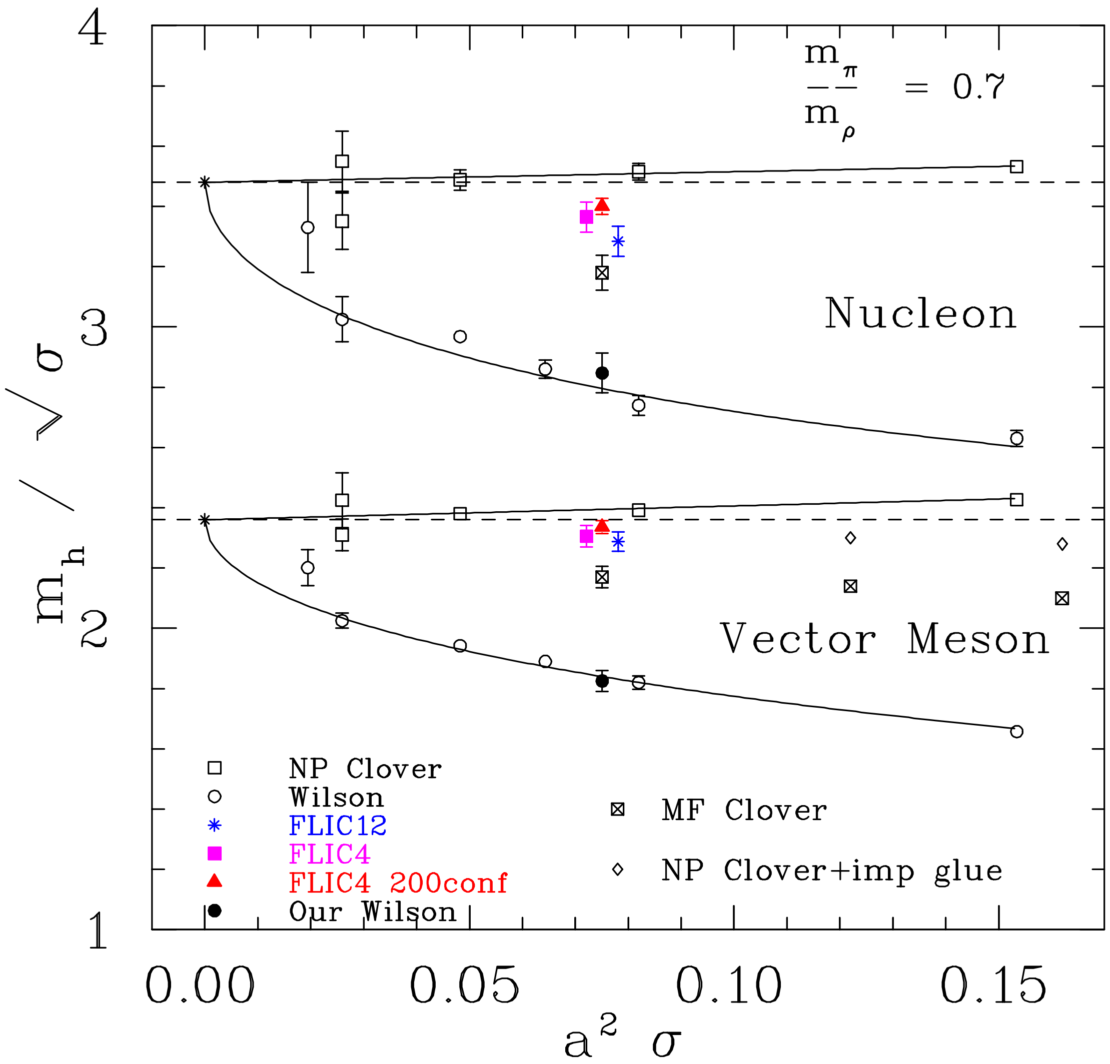

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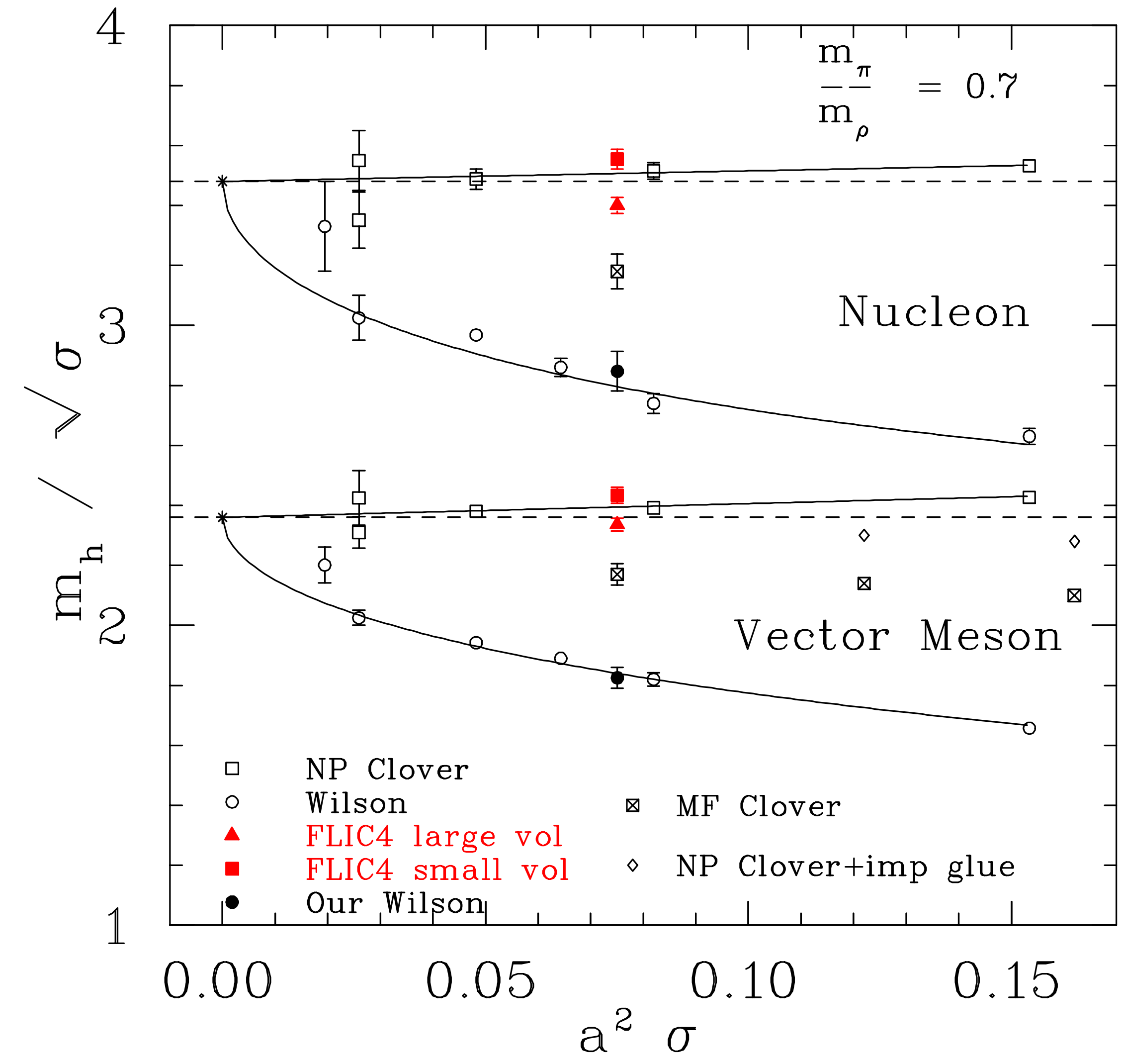

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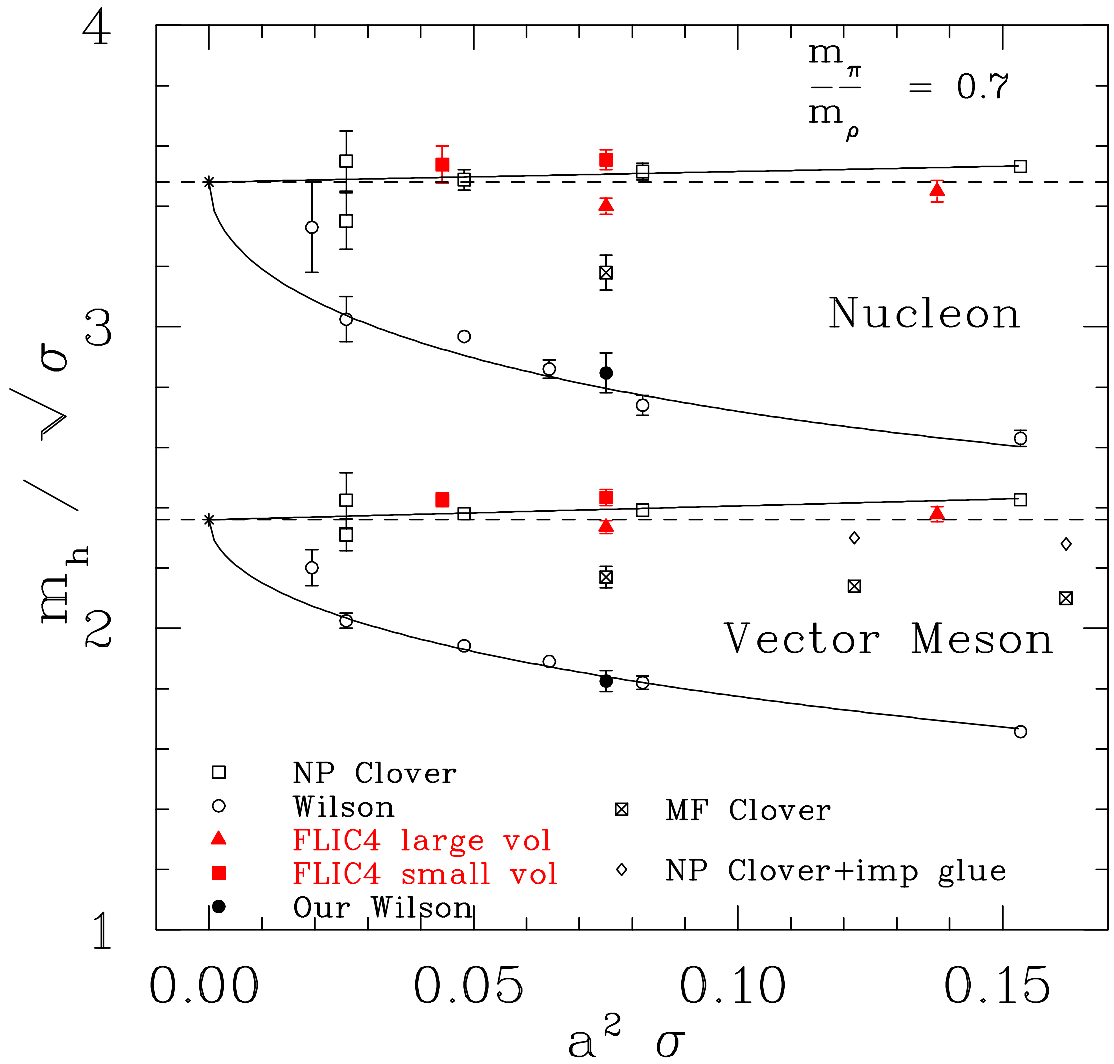

Light Quark Masses and Exceptional Configurations

- Chiral symmetry breaking in the action allows continuum zero modes of the Dirac operator to be shifted into the negative mass region.
- The quark propagator encounters singular behaviour
  - as the quark mass becomes light,
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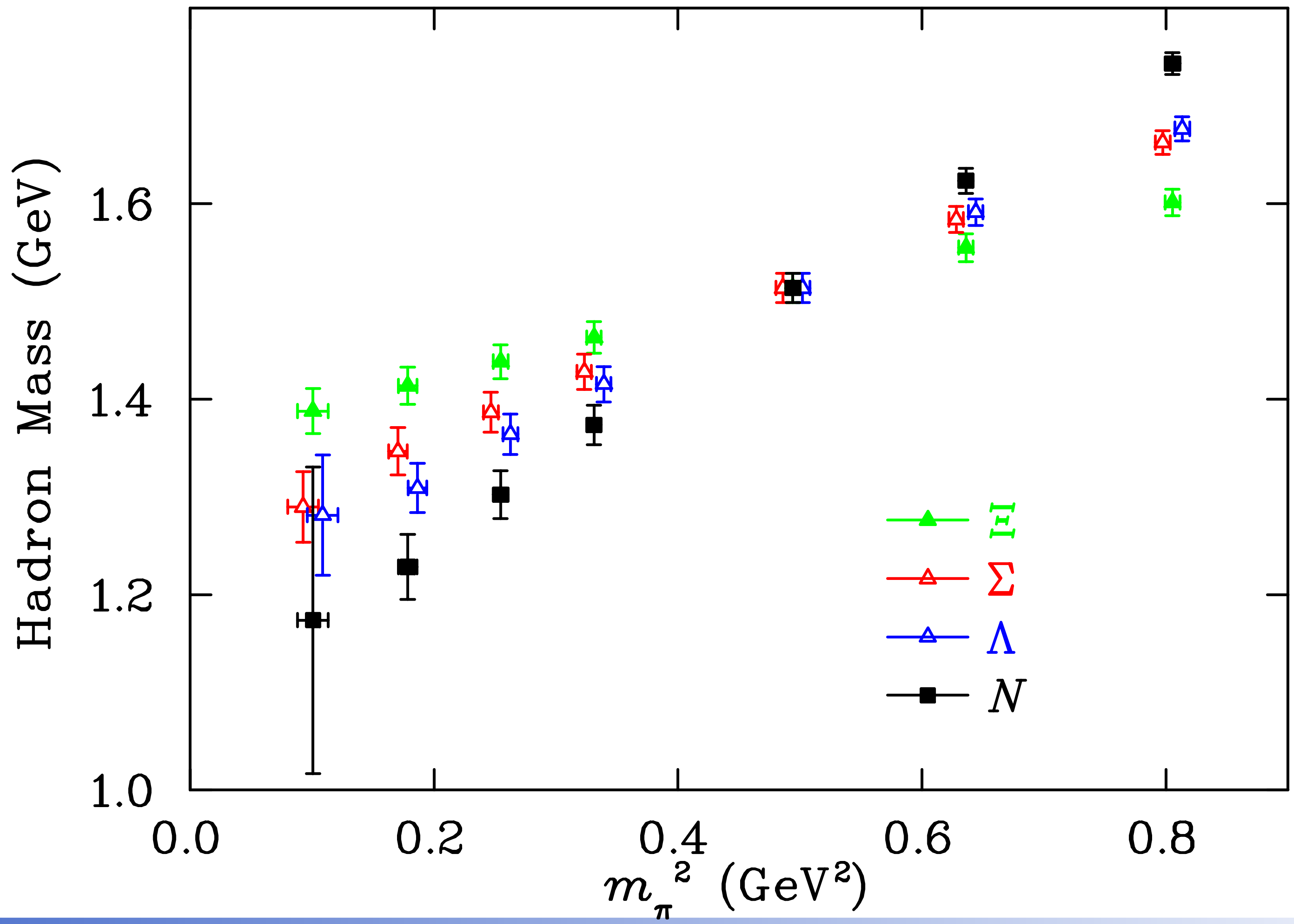
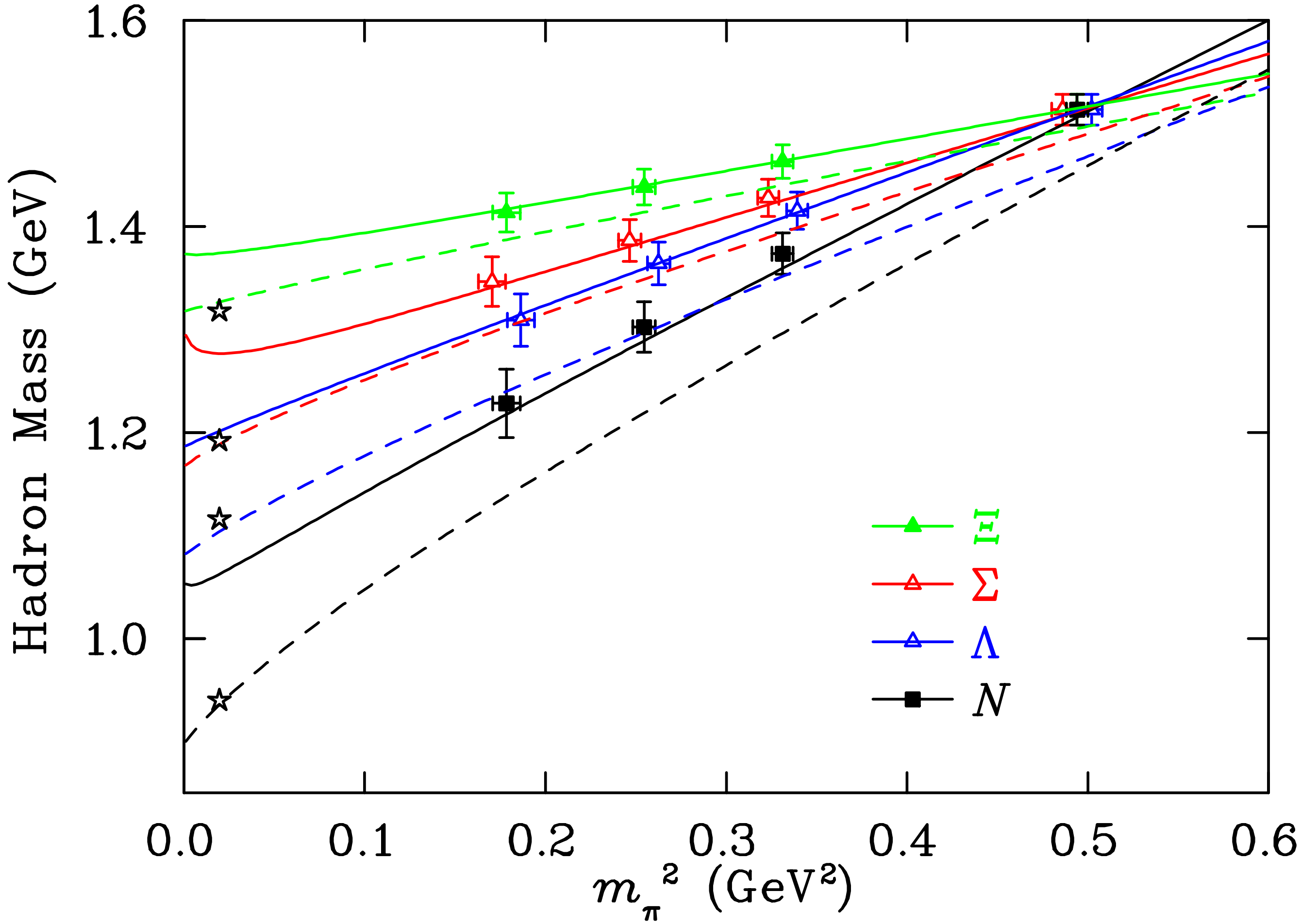
- Twisted mass [hep-lat/0111048] reached $m_\pi / m_\rho = 0.47$ on a fine lattice (0.1fm)

Octet Baryons with Light Quark Masses



Chiral extrapolation (incl unquenching)

Estimate of physical limit using chiral extrapolation and unquenching phenomenology R. Young, D. Leinweber, A. Thomas, *et al.*



Current Simulations

- $20^3 \times 40$ at $a = 0.134$ fm
- FLIC6 with 5-loop improved $F_{\mu\nu}$

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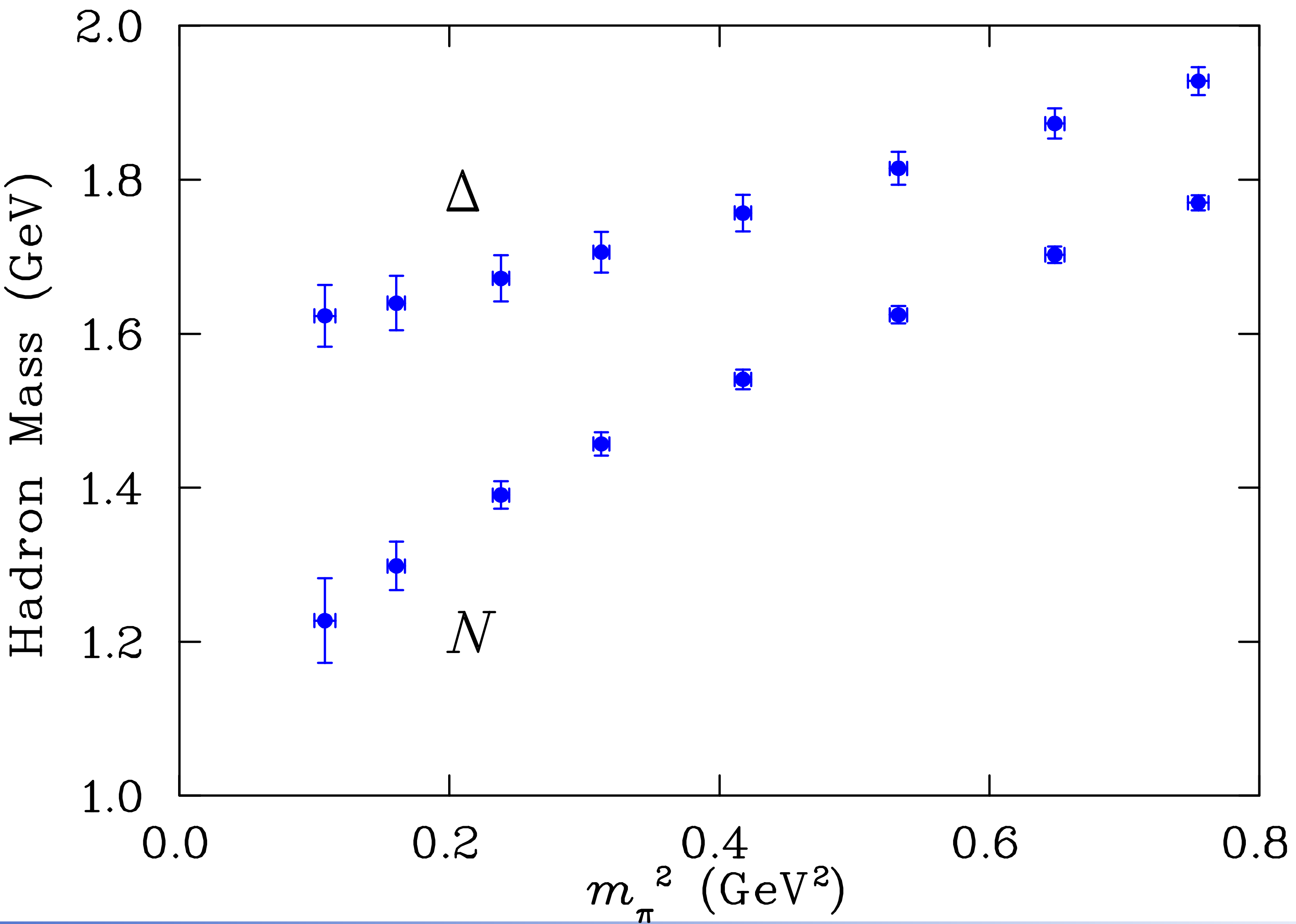
- $20^3 \times 40$ at $a = 0.134$ fm
- FLIC6 with 5-loop improved $F_{\mu\nu}$
- 8 masses

$$\frac{m_\pi}{m_\rho} = 0.75 \Rightarrow \frac{m_\pi}{m_\rho} = 0.35$$

- Currently 70 configurations $\longrightarrow$ 200

New Simulations with Light Quark Masses



Spectroscopy Issues

- Why are lowest positive parity (Roper) excitations

$$N^{1/2+}(1440),\ \Delta^{3/2+}(1600),\ \Sigma^{1/2+}(1690),\ \ldots$$

lighter than the lowest negative parity states?

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- What is the nature of $\Lambda^{1/2-}(1405)$?
  $\longrightarrow$ $\Sigma\pi$ channel coupling?
- What causes mass splitting between $\Lambda^{1/2^-}(1405)$ and $\Lambda^{3/2^-}(1520)$?

Survey of N^* Calculations

- Leinweber

  Phys. Rev. D **51** (1995) 6383

  $N'(1/2^+)$, Wilson fermions, OPE-based spectral ansatz

- Lee & Leinweber

  Nucl.Phys.B (Proc.Suppl.) **73** (1999) 258

  $N^*(1/2^-), N^*(3/2^-)$, $\mathcal{O}(a^2)$ tree level tadpole-improved Dχ34 action.

- Lee

  Nucl.Phys.B (Proc.Suppl.) **94** (2001) 251

  $N'(1/2^+), N^*(1/2^-)$, anisotropic improved gauge action ($a_s = 0.24$ fm), $D234$ quark action

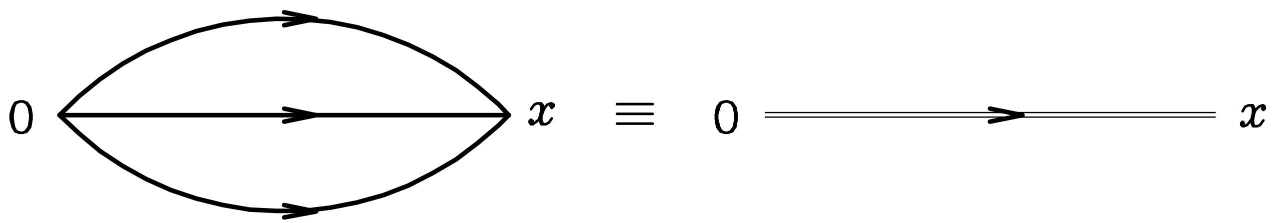
Survey of N^* Calculations

- Sasaki, Blum & Ohta
  hep-lat/0102010, hep-ph/0004252
  $N'(1/2^+), N^*(1/2^-)$, domain wall fermions
- Richards, hep-lat/0011025, Göckeler et al. hep-lat/0106022
  $N^*(1/2^-), \Delta^*(3/2^-)$, non-perturbatively improved clover quark action
- Nakajima, Matsufuru, Nemoto & Suganuma, hep-lat/0204014,
  $\Lambda^*_{\mathbf{1},\mathbf{8}}(1/2^-)$, anisotropic improved gauge action, $\mathcal{O}(a)$ improved quark action

Baryon Masses on the Lattice

- Two-point baryon correlation function:

$$G_{\alpha\alpha'}(t,\vec{p}) \equiv \sum_{\vec{x}} \exp(-i\vec{p}\cdot\vec{x}) \, \langle 0 \,| T \; \chi_\alpha(x) \; \bar{\chi}_{\alpha'}(0) | \, 0 \rangle$$



- χ is a baryon interpolating field, α, α' are Dirac indices.

Baryon Masses on the Lattice

- For large Euclidean time $t_E \to \infty$,

$$G_{\alpha\alpha'} = \frac{\lambda_{B+}^2}{2E_{B+}} (\gamma \cdot p + M_{B+})_{\alpha\alpha'} \, e^{-E_{B+}\, t} + \frac{\lambda_{B-}^2}{2E_{B-}} (\gamma \cdot p - M_{B-})_{\alpha\alpha'} \, e^{-E_{B-}\, t} \, .$$

- Periodic boundary conditions in the spatial directions.
- Fixed boundary condition in the time direction,

$$U_t(\vec{x}, nt) = 0 \, , \quad \forall \, \vec{x} \, .$$

Baryon Masses on the Lattice

- For large Euclidean time $t_E \to \infty$,

$$
\begin{aligned}
G_{\alpha\alpha'} &= \frac{\lambda_{B+}^2}{2E_{B+}} (\gamma \cdot p + M_{B+})_{\alpha\alpha'}\, e^{-E_{B+}\, t} \\
&+ \frac{\lambda_{B-}^2}{2E_{B-}} (\gamma \cdot p - M_{B-})_{\alpha\alpha'}\, e^{-E_{B-}\, t} .
\end{aligned}
$$

- Project positive or negative parity masses by taking trace of G with

$$
\Gamma_{\pm} = \frac{1}{2}(1 \pm \gamma_4) = \begin{pmatrix} \mathrm{I}\ (0) & 0 \\ 0 & 0\ (\mathrm{I}) \end{pmatrix} .
$$

Interpolating Fields

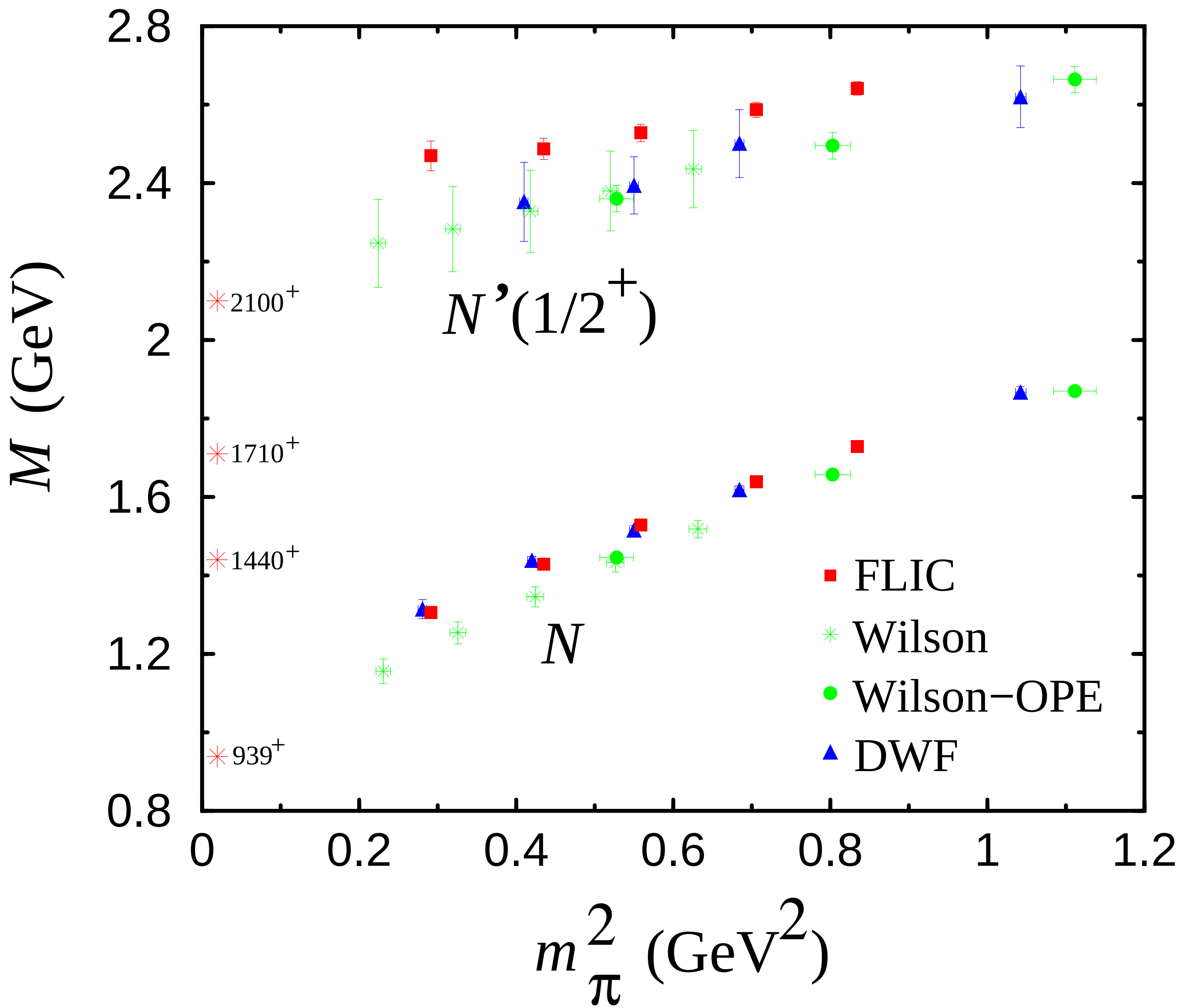
- Positive parity proton interpolating fields:

$$\chi_1^{p+}(x) = \epsilon^{abc}\left(u^{Ta}(x)\, C\gamma_5\, d^b(x)\right)\, u^c(x)$$
$$\chi_2^{p+}(x) = \epsilon^{abc}\left(u^{Ta}(x)\, C\, d^b(x)\right)\, \gamma_5\, u^c(x)$$

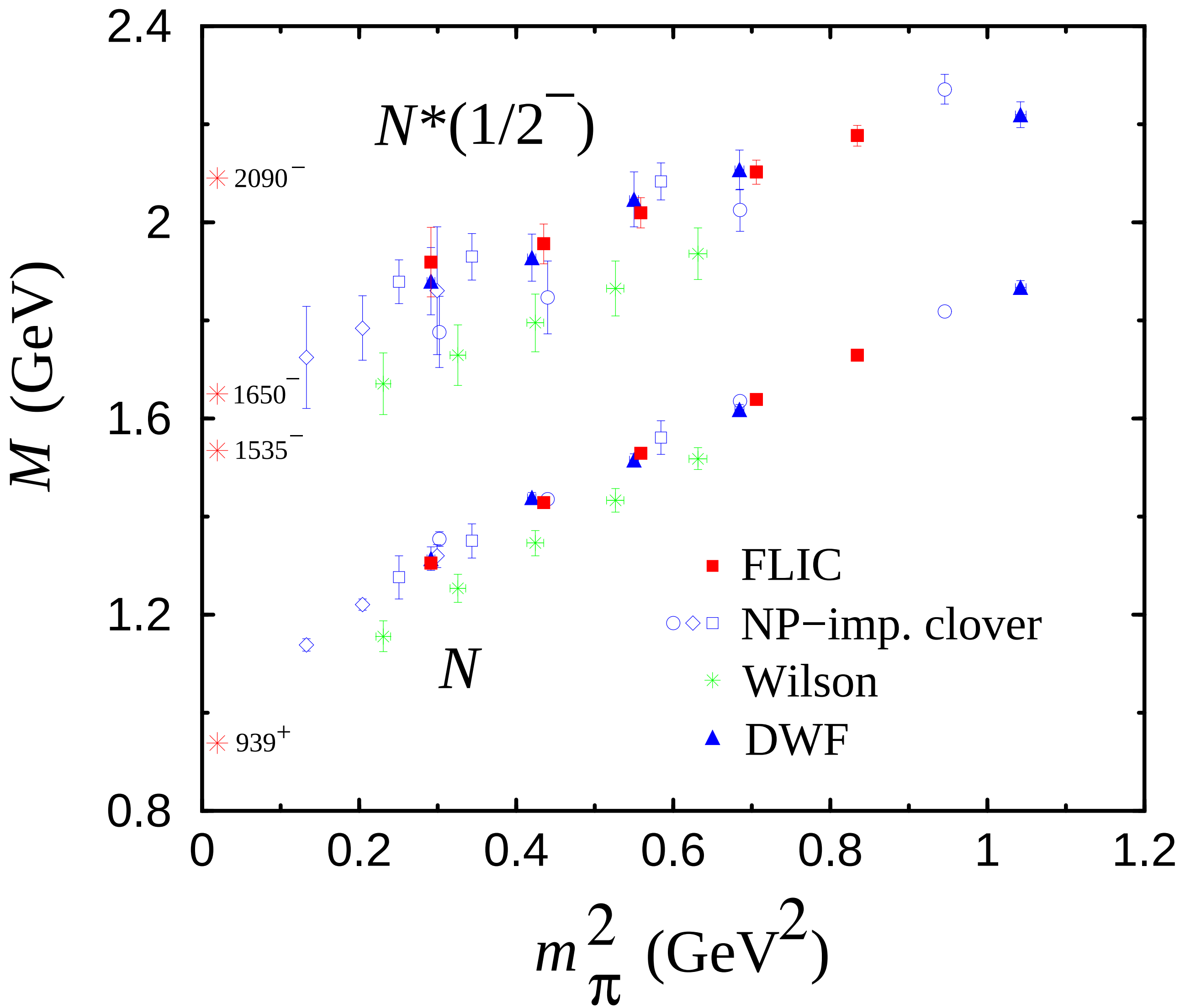
a, b, c are colour indices

- χ_1 involves *upper* $\times$ *upper* $\times$ *upper* components.
$\longrightarrow$ "diquark" $(u^T \cdots d)$ couples to spin 0 (attractive).
$\longrightarrow$ χ_1 is $\mathcal{O}(1)$ in the nonrelativistic limit.
- χ_2 mixes *upper* $\times$ *lower* $\times$ *lower* components.
$\longrightarrow$ $\vec{\sigma} \cdot \vec{p} \Rightarrow$ relative $\ell = 1$, "diquark"
$\longrightarrow$ $\chi_2 = \mathcal{O}(p^2/E^2)$, vanishes in the nonrelativistic limit.
$\longrightarrow$ Known to have little overlap with ground state.

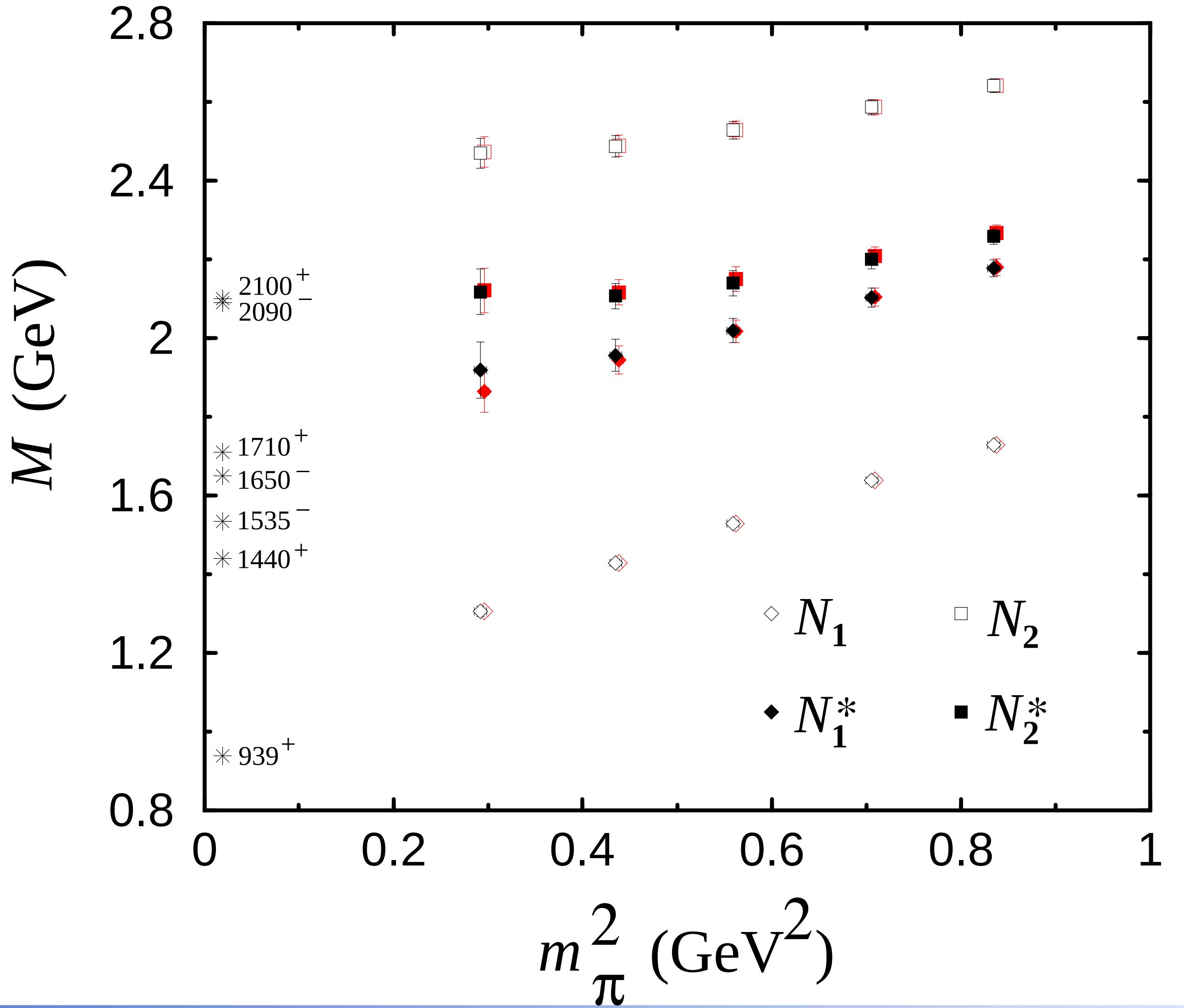
$N'(1/2^+)$ Mass from the χ_2 Correlation Function



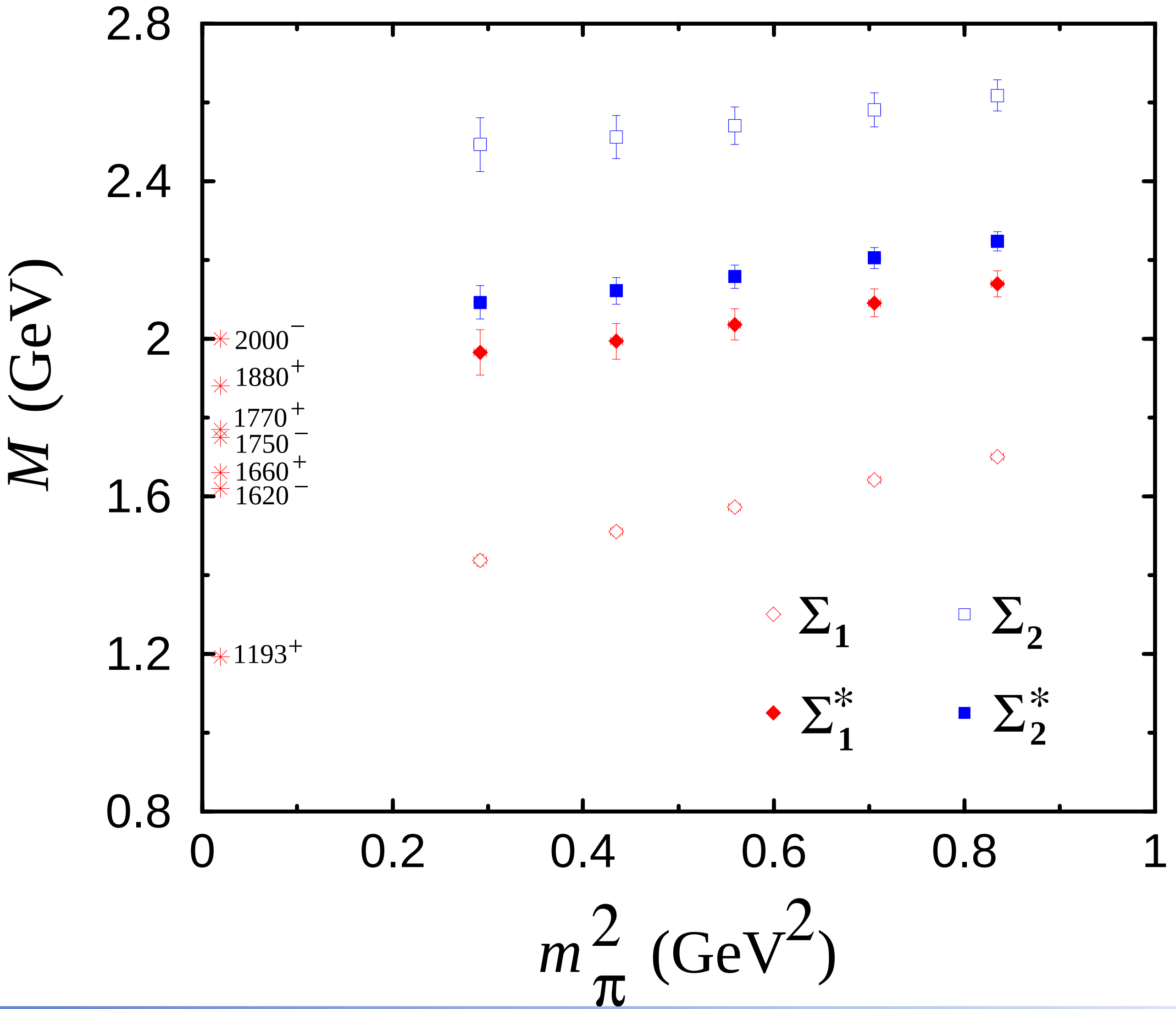
$N^*(1/2^-)$ Mass from the χ_1 Correlation Function



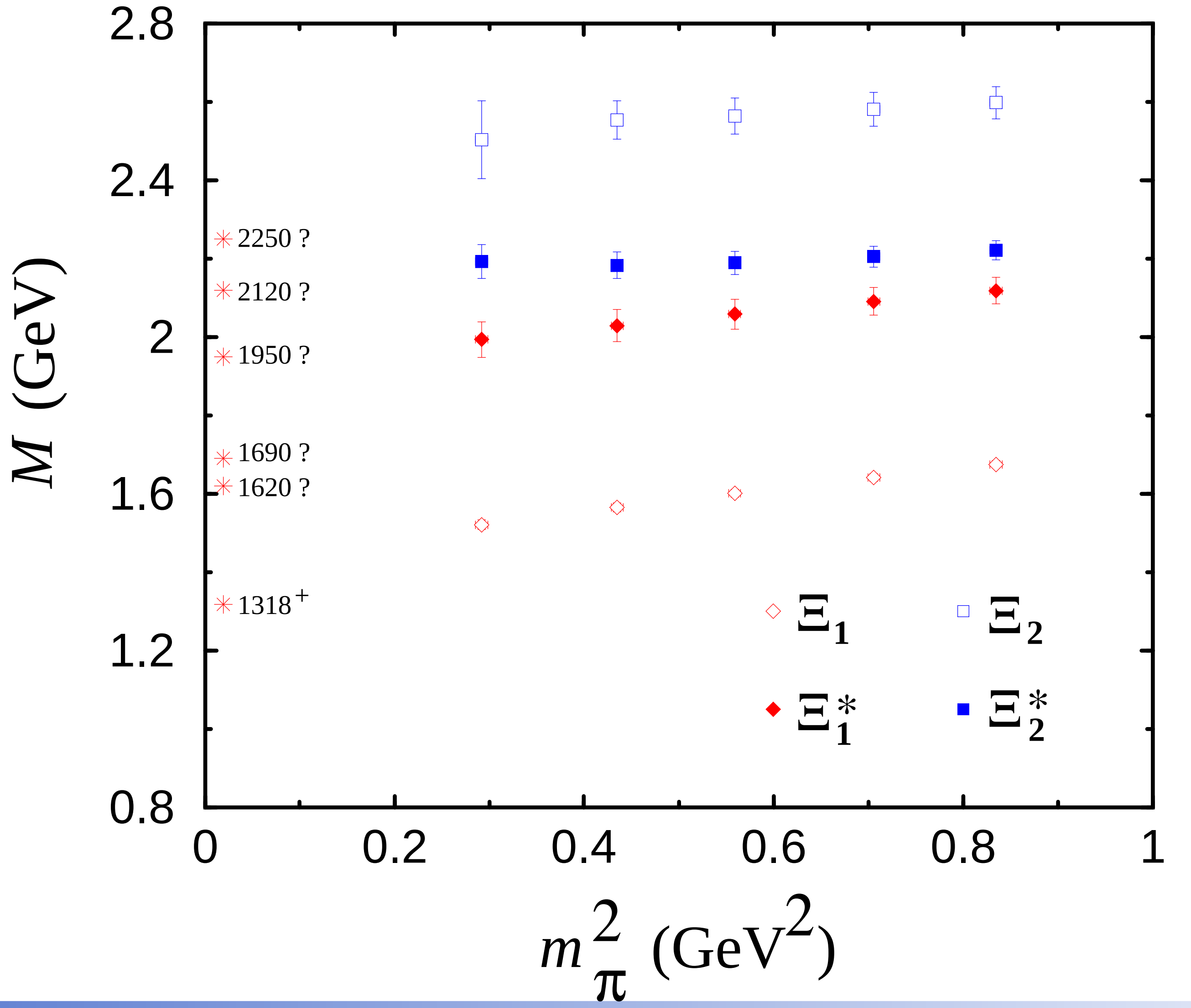
Summary of Nucleon Results



Summary of Σ Results



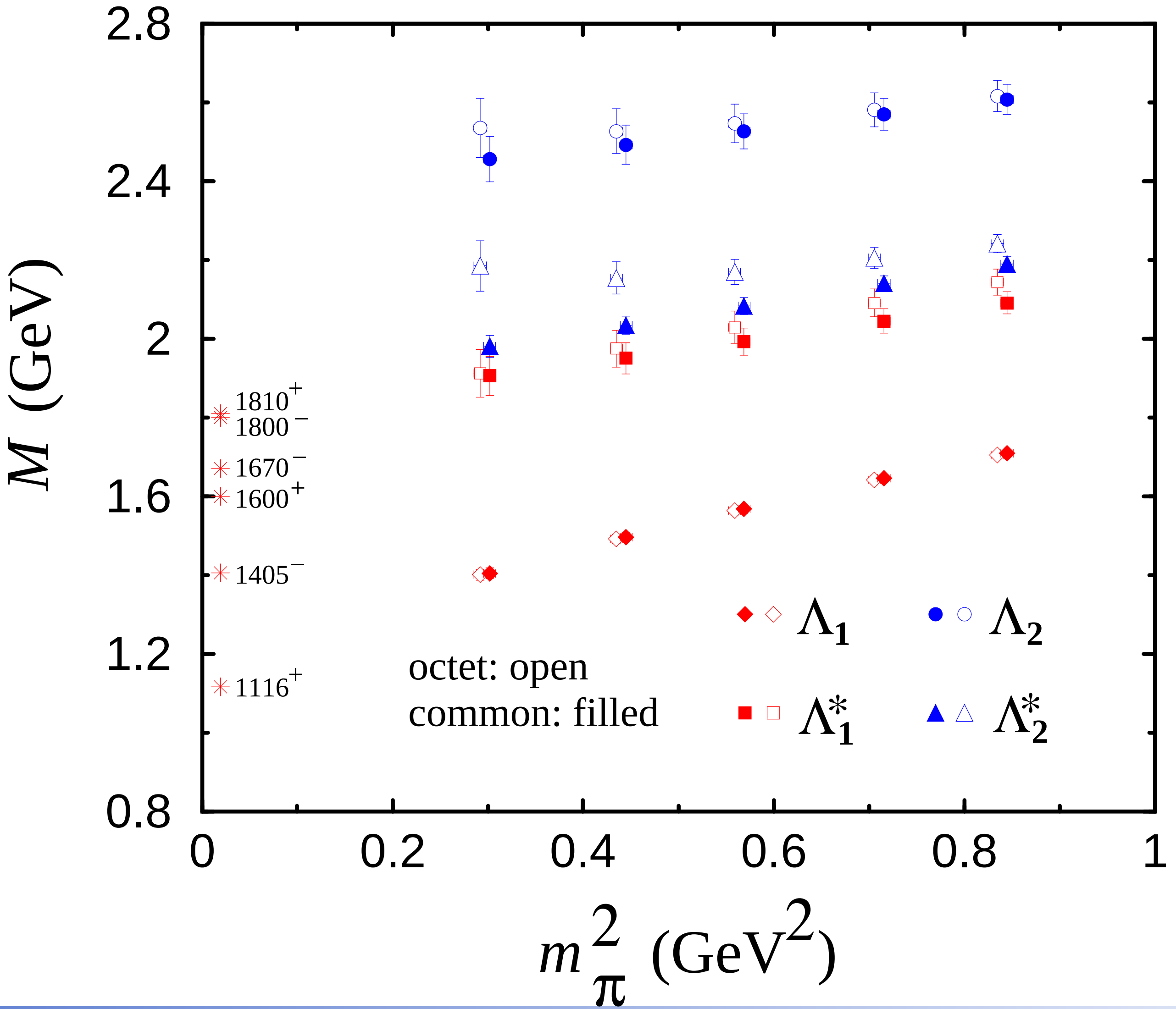
Summary of Ξ Results



Λ Interpolating fields

- We consider:
- The full SU(3)-flavour octet Λ interpolating field, and
- The SU(2)-isospin singlet Λ interpolating field.
- The latter contains terms Common to both the
  - SU(3)-flavour octet interpolator, and
  - SU(3)-flavour singlet interpolator.
- No SU(3)-flavour representation bias in Λ-common.

Λ Summary



2.8
2.4
2
1.6
1.2
0.8
0
0.2
0.4
0.6
0.8
1
M (GeV)
m_π^2 (GeV^2)
1810$^+$
1800$^-$
1670$^-$
1600$^+$
1405$^-$
1116$^+$
octet: open
common: filled
Λ_1
Λ_2
Λ^*_1
Λ^*_2

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- No evidence of Roper states using conventional interpolators.
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- Good agreement among calculations of $N^* 1/2^-$ using improved actions.
- Resolved clear splitting between $N_1^* 1/2^-$ and $N_2^* 1/2^-$.
- No evidence for the mass suppression of $\Lambda^*(1405)$.
  - Suppression of meson-cloud in Quenched Approximation?
  - Are exotic interpolating fields required?

Spin-3/2 Interpolating Fields

$$\chi_\mu^{3/2}(x) = \epsilon^{abc} \left(u^{Ta}(x) C \gamma_5 \gamma_\mu d^b(x) \right) \gamma_5 u^c(x) \, .$$

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$$\chi_\mu^{3/2}(x) = \epsilon^{abc}\left(u^{Ta}(x) C\gamma_5\gamma_\mu d^b(x)\right)\gamma_5 u^c(x)\,.$$

- Apply the spin-3/2 projection operator

$$P_{\mu\nu}^{3/2}(p) = g_{\mu\nu} - \frac{1}{3}\gamma_\mu\gamma_\nu - \frac{1}{3p^2}\left(\gamma\cdot p\,\gamma_\mu\, p_\nu + p_\mu\,\gamma_\nu\,\gamma\cdot p\right).$$

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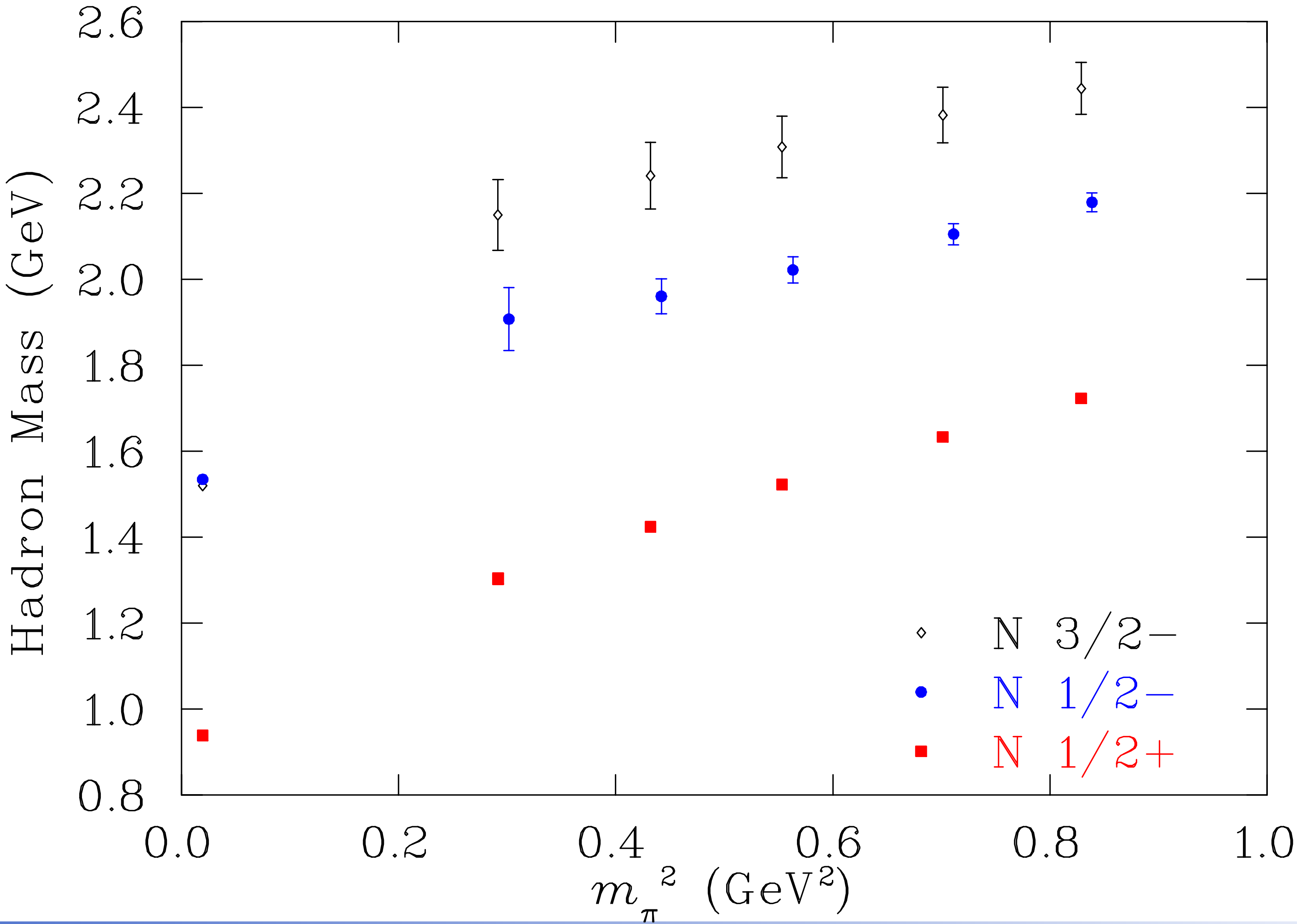
- Computational cost is 64 times the proton correlation function.
- Spin projection followed by parity projection allows isolation of 4 states.

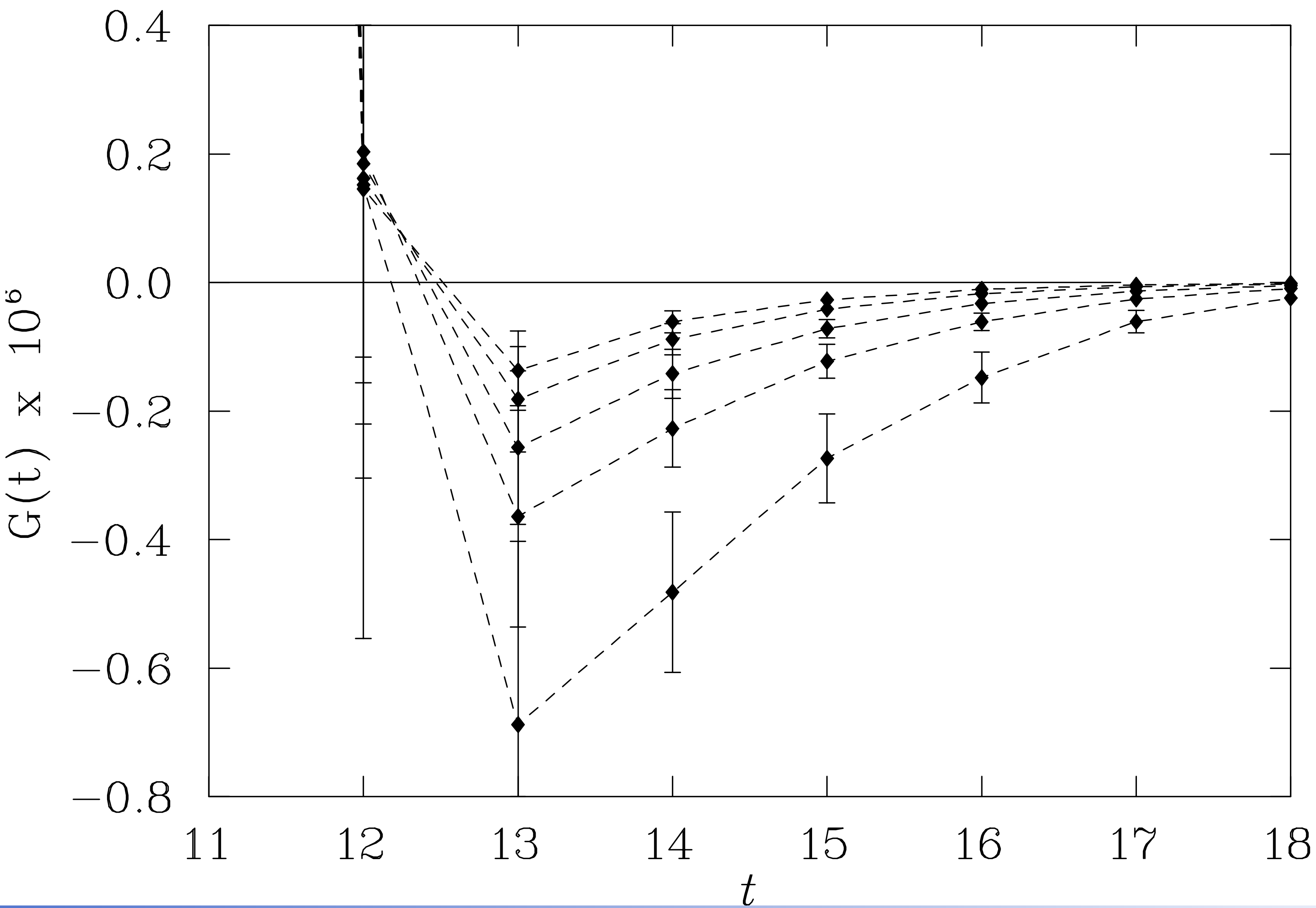
$$N^*1/2^+\,,\;N^*1/2^-\,,\;N^*3/2^+\,,\;N^*3/2^-\,.$$

$N^{*}3/2^{-}$



Hadron Mass (GeV)
2.6
2.4
2.2
2.0
1.8
1.6
1.4
1.2
1.0
0.8
0.0
0.2
0.4
0.6
0.8
1.0
m_π^2 (GeV^2)
N 3/2−
N 1/2−
N 1/2+

Quenched p-wave $N\,\eta'$ Negative Metric Contrib. to $N^*3/2^+$



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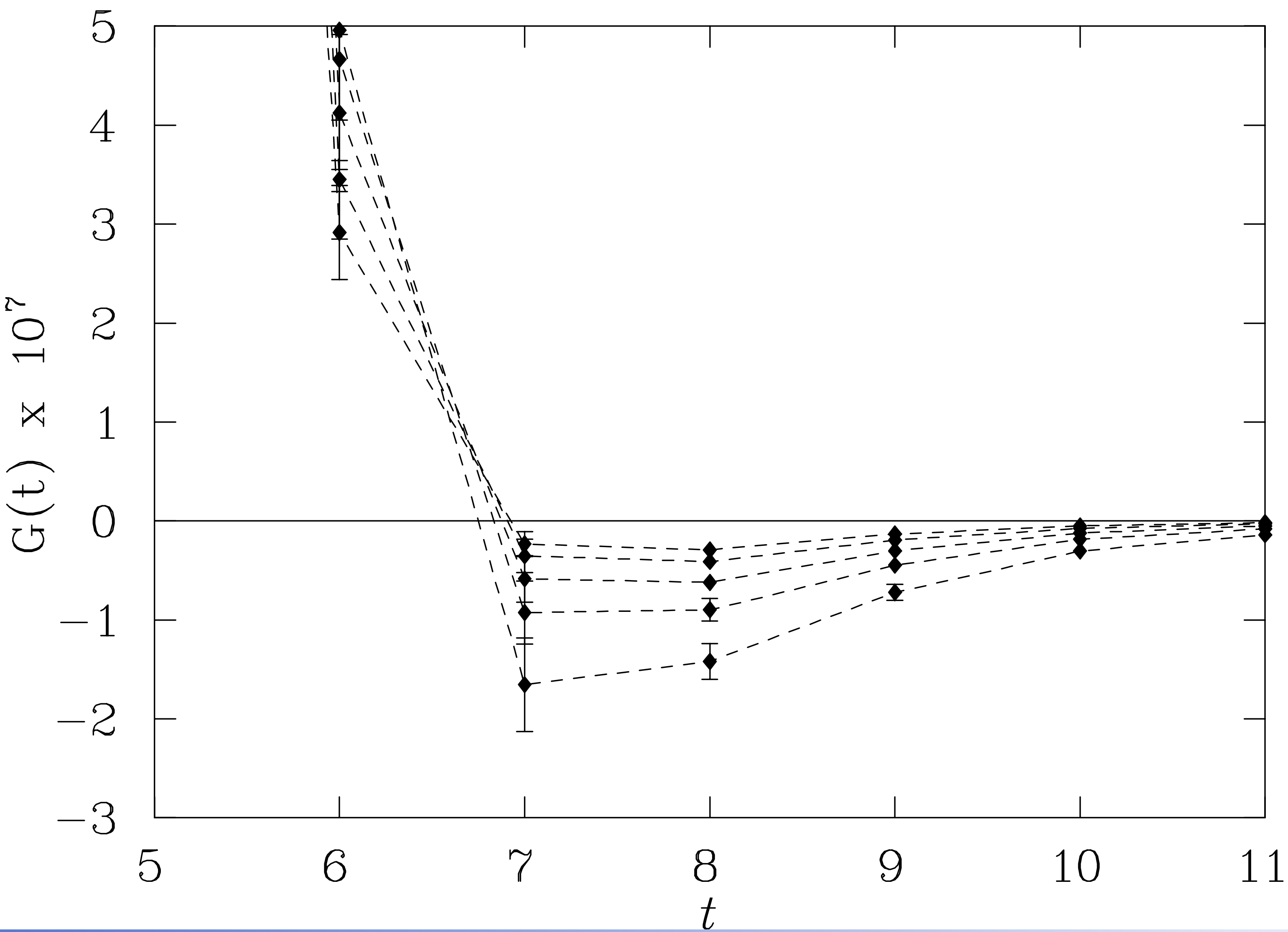
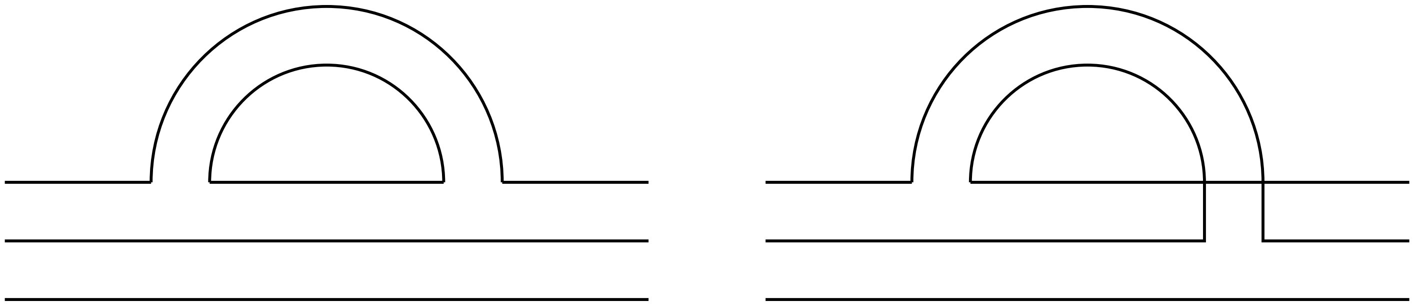


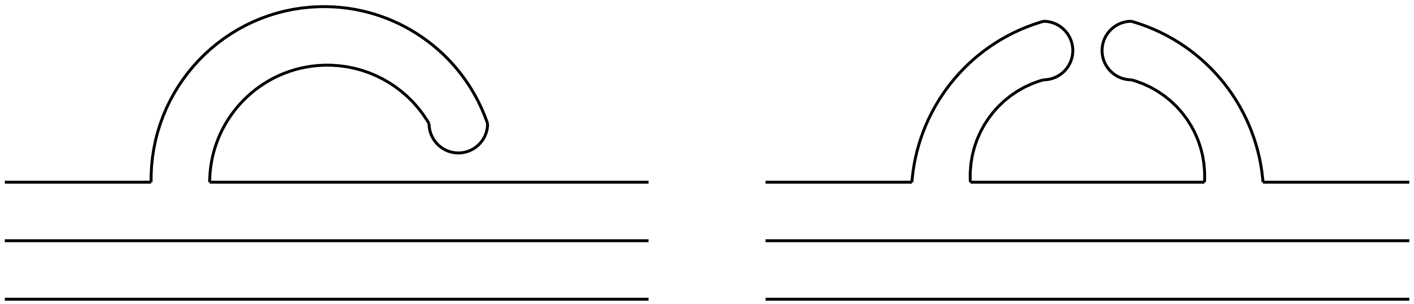
Quenched Chiral Nonanalytic Behavior

- Conventional mesons are modified



(a) (b)

- The η' remains degenerate with the pion and can contribute negatively to correlation functions.



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The Roper in Quenched QCD

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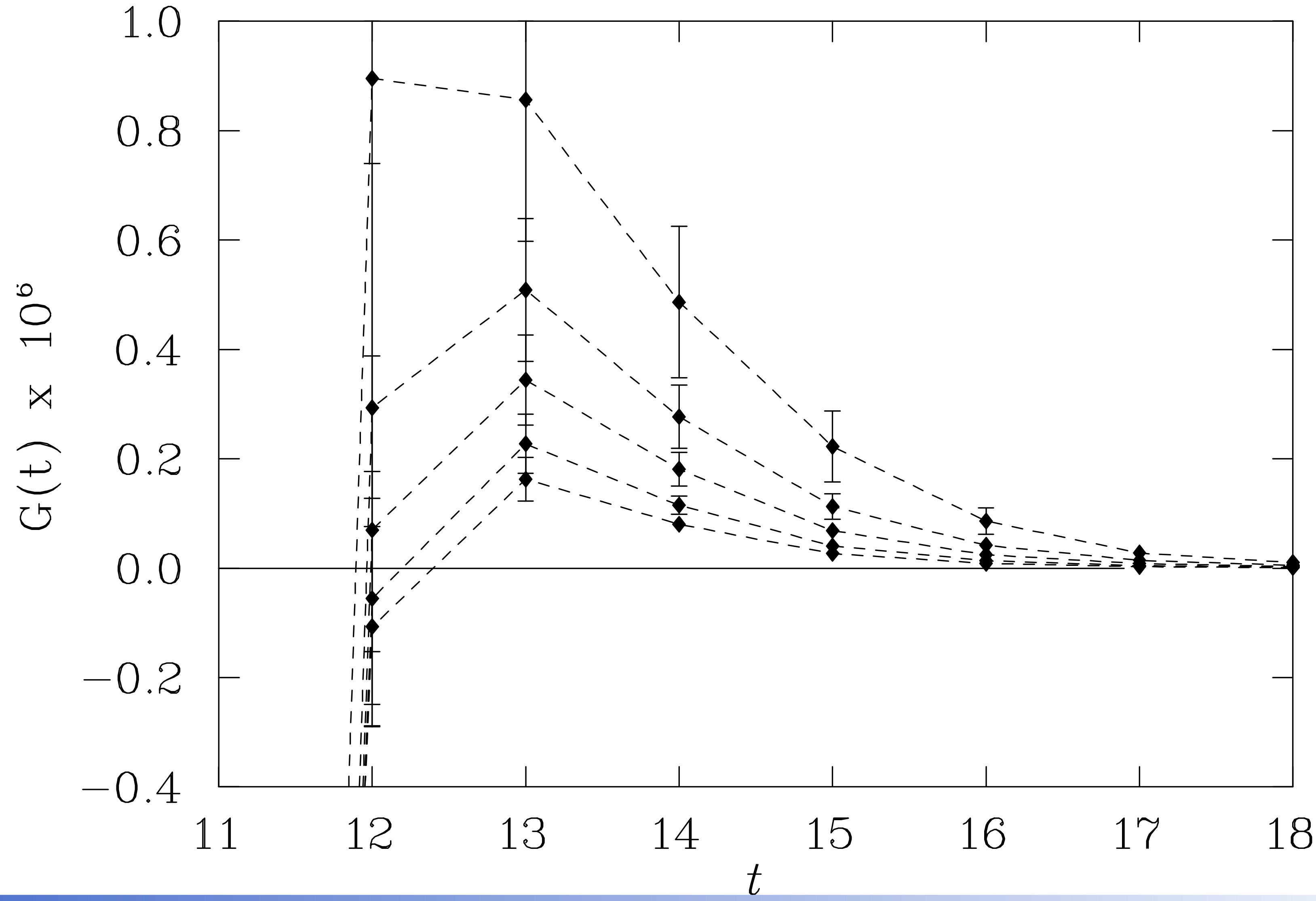
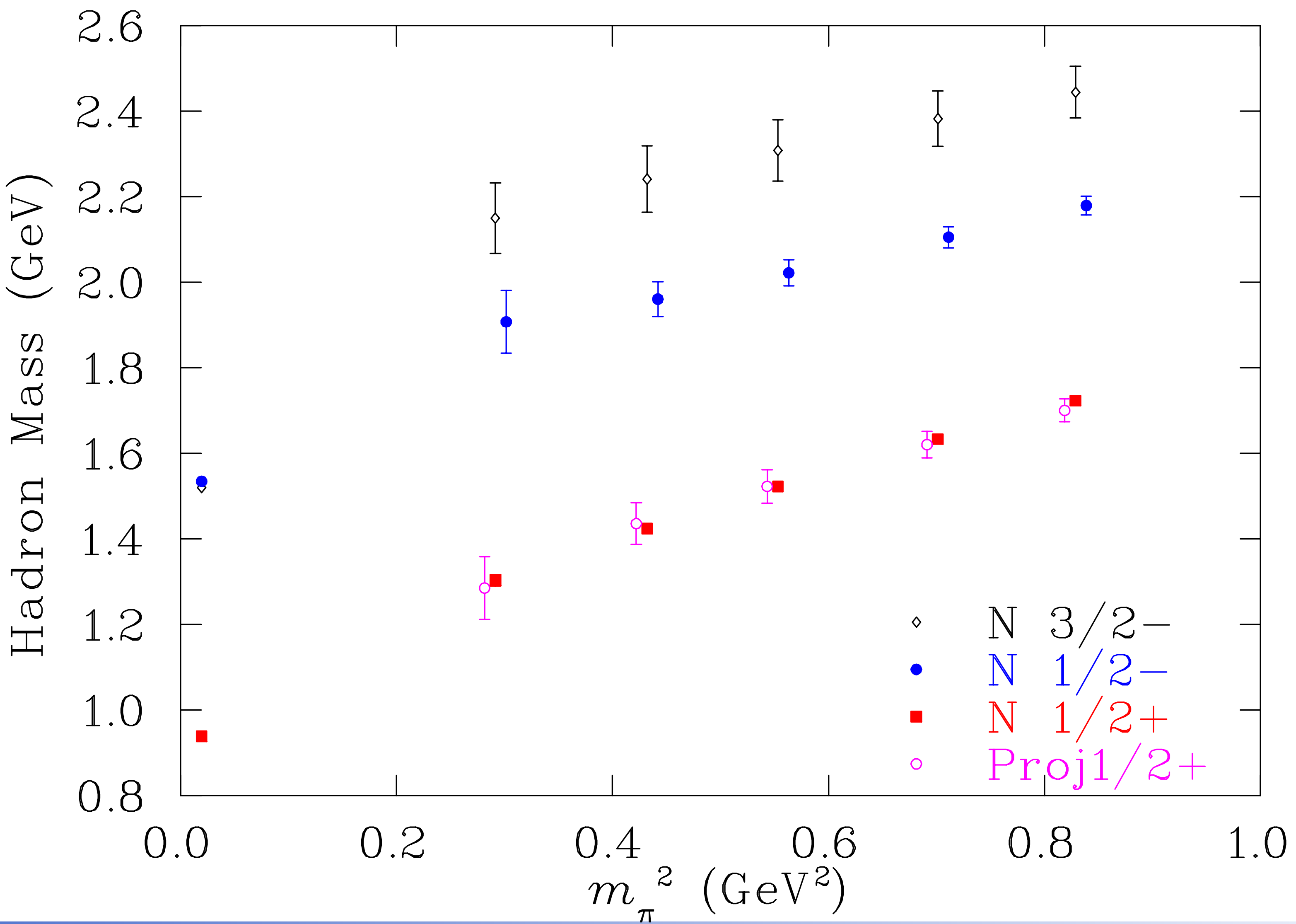
- Ground state nucleon is also $J^P = 1/2^+$.
- If $N\,\eta'$ dominates the correlation function function at intermediate Euclidean time,
  - Correlation function will be negative and change sign as the ground state nucleon eventually dominates the correlation function.

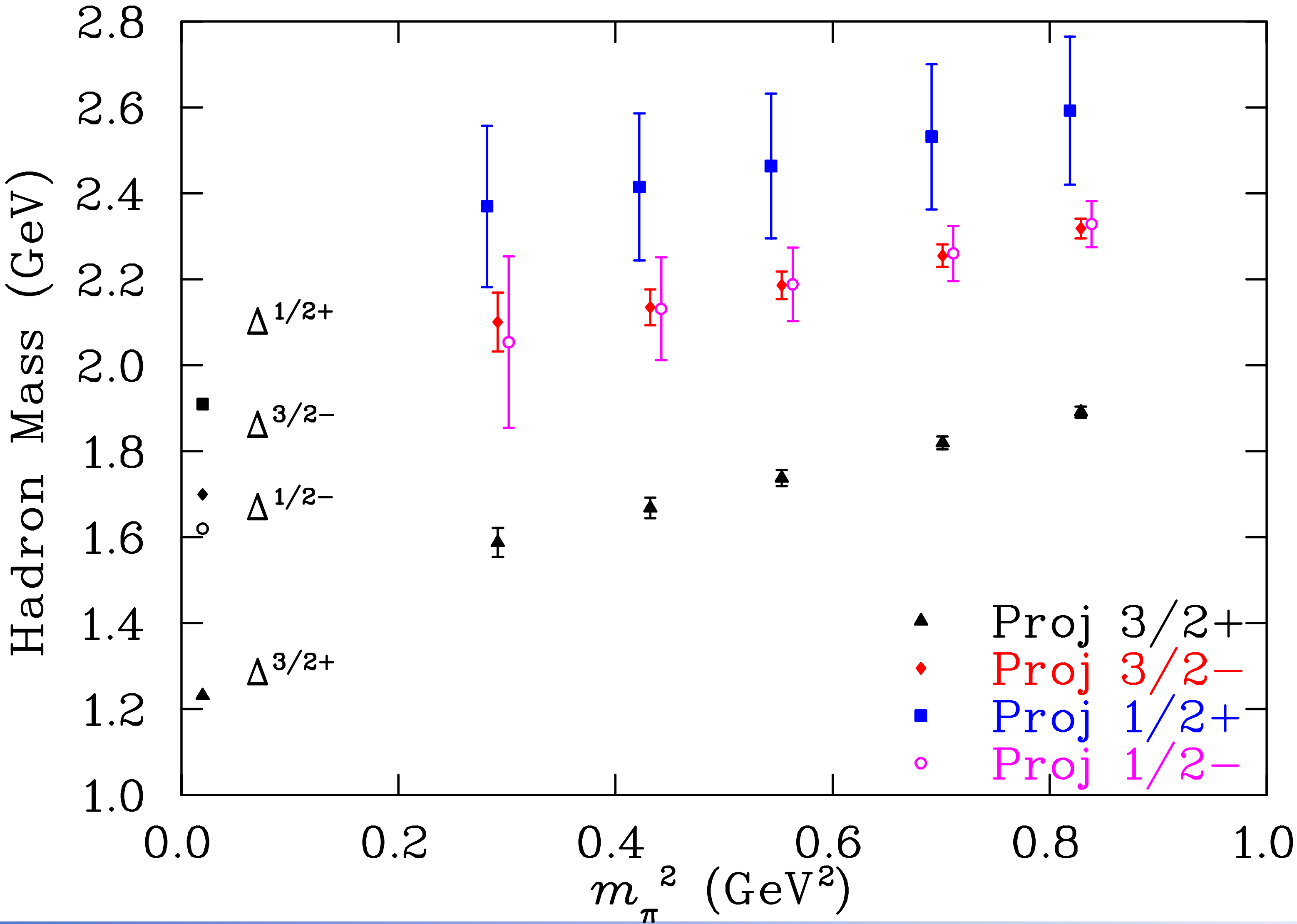
Quenched p-wave $N\,\eta'$ Negative Metric Contrib. to $N^*1/2^+$



$N^*3/2^-$ and $N1/2^+$ from Spin-3/2 Interpolating Field



$\Delta 3/2^+$, $\Delta^* 3/2^-$, $\Delta^* 1/2^\pm$



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- Resolved all four Δ states

$$\Delta^3/2^+,\ \Delta^*3/2^-,\ \Delta^*1/2^+\ \Delta^*1/2^-$$

FLIC Conclusions & Future Work

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  $\longrightarrow$ Mesons with Exotic Quantum Numbers
- Three-point correlation functions
  $\longrightarrow$ Electromagnetic Form Factors
  $\longrightarrow$ $N \rightarrow \Delta$ transition form factors
  $\longrightarrow$ $N \rightarrow N^*$ transition form factors

Baryon Masses on the Lattice

- Two-point baryon correlation function:

$$G_{\alpha\alpha'}(t,\vec{p}) \equiv \sum_{\vec{x}} \exp(-i\vec{p}\cdot\vec{x})\,\langle 0\,|T\,\chi_\alpha(x)\,\bar{\chi}_{\alpha'}(0)|\,0\rangle$$

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χ is a baryon interpolating field and α, α' are Dirac indices

- Overlap of χ with positive or negative parity states $|B^\pm\rangle$ parameterized by coupling strength $\lambda_{B^\pm}$:

$$\begin{aligned} \langle 0\,|\,\chi(0)\,|\,B^+(p,s)\rangle &= \lambda_{B^+}\sqrt{\frac{M_{B^+}}{E_{B^+}}}\,u_{B^+}(p,s) \\ \langle 0\,|\,\chi(0)\,|\,B^-(p,s)\rangle &= \lambda_{B^-}\sqrt{\frac{M_{B^-}}{E_{B^-}}}\,\gamma_5 u_{B^-}(p,s) \end{aligned}$$

Exotic Quantum Numbers

- A $\overline{q}q$ system is an eigenstate of parity with $P = (-1)^{L+1}$
- Charge conjugation applied to a neutral system provides $C = (-1)^{L+S}$
- For $S = 0$, the total angular momentum $J = L$ and $CP = -1$. $J^{PC} = 0^{-+},\ 1^{+-},\ 2^{-+}, \ldots$
- One cannot form the alternate $CP = -1$ states $J^{PC} = 0^{+-},\ 1^{-+},\ 2^{+-}, \ldots$ and these states are known as *exotics*.
- These states can be created on the lattice via *hybrid operators*.

Hybrid Meson Interpolators

- Combine the following ingredients:
  - $\bar{q}^a \Gamma q^a$: colour singlet quark bilinear
  - $\bar{q}^a \Gamma q^b$: colour octet quark bilinear
  - B^{ab}: colour magnetic field, colour 8, $J^{PC} = 1^{+-}$
  - E^{ab}: colour electric field, colour 8, $J^{PC} = 1^{--}$
- Pion plus color magnetic field
  - $0^{-+} \otimes 1^{+-} = 1^{--}$
  - 1^{--}: $\bar{q}^a \gamma_5 q^b B_i^{ab}$ (ρ meson).
- Rho plus color magnetic field
  - $1^{--} \otimes 1^{+-} = 0^{-+} \oplus 1^{-+} \oplus 2^{-+}$
  - 0^{-+}: $\bar{q}^a \gamma_i q^b B_i^{ab}$ (π meson)
  - 1^{-+}: $\epsilon_{ijk} \bar{q}^a \gamma_i q^b B_j^{ab}$ **(exotic)**