Physics of the Light-Cone Vacuum

Content

Introduction

Characteristic for light-cone kinematics is the divergence of the energy of a particle with decreasing light-cone momentum

$$\lim_{k_- \to 0} k_+ = \lim_{k_- \to 0} \frac{m^2 + k_\perp^2}{2k_-} \to \infty$$

As a consequence, large excitation energies are associated with long-wavelength phenomena - unique for physics on the light-cone. Consequences of the occurrence of infinite excitation energies is the subject of this talk.

Divergence makes regularization in addition to the more standard short-wavelength regularization of the quantum fluctuations necessary. Regularization in calculation of long-wavelength properties already required at level of non-interacting theories. 2-point function of a free scalar field displays the subtleties of light-cone physics

$$-i\langle 0|T(\varphi(z)\varphi(0))|0\rangle = D(z) = \int \frac{d^4k}{(2\pi)^4} \frac{e^{ikz}}{k^2 - m^2 + i\epsilon}$$

$$= -i \int \frac{d^2k_\perp}{(2\pi)^3} \int_0^\infty \frac{dk_-}{k_-} e^{-i\frac{k_\perp^2 + m^2 - i\epsilon}{2k_-}|z^+| + ik_\perp z^\perp - ik_- z^- \epsilon(z^+)}$$

$$= \frac{1}{2\pi^2} \frac{m}{\sqrt{(-z)^2}} K_1(m\sqrt{(-z)^2})$$

In ordinary coordinates for spacelike z

$$\lim_{z^0 \to 0} < 0|T(\varphi(z)\varphi(0))|0 > \Big|_{z^2} =< 0|\varphi^\dagger(z^0 = 0^+, \mathbf{z})\varphi(0)|0 > \Big|_{\mathbf{z}^2 = -z^2}$$

the vev of Heisenberg operators agrees in the limit of $z^0 \to 0$ with the corresponding Schrödinger operators. In light-cone quantization this is not the case. Limit of vev of Heisenberg operators

$$\lim_{z^+ \to 0} D(z) = \frac{1}{2\pi^2} \frac{m}{\sqrt{\mathbf{z}_\perp^2}} K_1(m\sqrt{\mathbf{z}_\perp^2})$$

different from vev of the corresponding Schrödinger operators

$$D(z^+ = 0, z^-, z^\perp) = -i\frac{1}{(2\pi)^3} \int d^2k_\perp e^{ik_\perp z^\perp} \int_0^\infty \frac{dk_-}{k_-} e^{ik_- z^-}$$

Difference arises from long-wavelength excitations $k_- \to 0$ which for $z^+ = 0$ contribute without any suppression. Dynamical quantities of physical relevance are associated with expectation values of Heisenberg operators. Schrödinger operators only see the "trivial" light-cone vacuum.

- Singularities at long-wavelength essential for description of condensates on light-cone. Singularities make short-time evolution non-perturbative and gives rise to significant differences between free-and interacting theories - Condensate in large N QCD_{1+1}.

- Physics of long-wavelength fluctuations probed in Casimir effect.
  Study of systems with finite size or finite temperature on light-cone.

Chiral Condensate in QCD $_{1+1}$*

Definition of condensate with respect to non-interacting theory

$$\langle\bar\psi\psi\rangle = \lim_{z\to 0}\left[\langle\bar\psi\psi\rangle_z - \langle\bar\psi\psi\rangle_z^0\right] ,$$

Regularization by point splitting

$$\langle\bar\psi\psi\rangle_z = \langle 0|\bar\psi(z)Pe^{ig\int_0^z dx^\mu A_\mu}\psi(0)|0\rangle$$

Gauge string required by gauge invariance.

$$\langle\bar\psi\psi\rangle_z = \langle 0|\bar\psi(0)W(z)\psi(0)|0\rangle$$

$$W(s) = e^{-iz^\mu(P_\mu - gA_\mu(0))}$$

Chiral condensate given by short-time evolution of a system of light quarks coupled to the current of an infinitely heavy quark

$$\delta H = \int d^3x j^\mu A_\mu = \int d^3x\delta(\mathbf{x})A_0(\mathbf{x},0)$$

* M. Burkardt and M. Thies

't Hooft model

Time evolution of quark propagator

$$\langle\bar{\psi}\psi\rangle_z = \langle 0|\bar{\psi}_i(0)(\mathrm{e}^{-\mathrm{i}z^+ H_{\mathrm{eff}}})_{ij}\psi_j(0)|0\rangle$$

Effective Hamiltonian in light-cone gauge ($A_- = 0$),

$$(H_{\mathrm{eff}})_{ij} = (P_+ + \lambda P_-)\delta_{ij} - g[A_+(0)]_{ij}.$$

Straight-line x(s), with slope $\lambda = z^-/z^+$
P_+ and P_- Hamiltonian and momentum operator of 't Hooft model

After Elimination

$$\psi(x) = \frac{1}{2^{1/4}}\begin{pmatrix} 1 \\ \frac{m}{\mathrm{i}\sqrt{2}\partial_-} \end{pmatrix}\varphi \ ,$$

of constrained spinor components

$$\begin{aligned}\langle\bar{\psi}\psi\rangle_z &= \int \frac{\mathrm{d}p}{2\pi}\frac{m}{p}C(p,z^+) \\ C(p,z^+) &= \int \frac{\mathrm{d}q}{2\pi}\langle 0|\varphi_i^\dagger(p)(\mathrm{e}^{-\mathrm{i}H_{\mathrm{eff}}\,z^+})_{ij}\varphi_j(q)|0\rangle \ .\end{aligned}$$

Dynamics in Large N-limit

- Triviality of Vacuum

$$\varphi_i^\dagger(p)\varphi_i(p') \to 2\pi\delta(p-p')N\theta(-p),$$

- Single Particle Properties

$$[P_+ + \lambda P_-]\varphi_i(p)|0\rangle = \left(\frac{m_r^2}{2p} + \lambda p\right)\varphi_i(p)|0\rangle,$$

with self-energy

$$m_r^2 \equiv m^2 - \frac{Ng^2}{2\pi}$$

- Gauge field generated by fermion color charge density

$$[A_+(0)]_{ij} = -\frac{g}{2}\int\frac{dp'}{2\pi}\frac{dp''}{2\pi}\frac{\varphi_j^\dagger(p')\varphi_i(p'')}{(p'-p'')^2}\ .$$

- Resulting Schrödinger type time evolution equation (units $Ng^2/2\pi = 1$)

$$
\begin{aligned}
\mathrm{i}\dot{C}(p,t) &= \left(\frac{m^2-1}{2p} + \lambda p\right) C(p,t) + \frac{1}{2p} C(0,t) \\
&- \frac{1}{2} \int_0^\infty \mathrm{d}p' \frac{1}{p'-p} \frac{\partial C(p',t)}{\partial p'} \ .
\end{aligned}
$$

- Initial condition

$$
C(p,0) = N
$$

Properties of Time-Evolution

Short-time evolution

$$\mathrm{i}\dot{C}(p,t=0) = N\left(\frac{m^2}{2p}+\lambda p\right)$$

Infrared singularity, characteristic for the short-time light-cone dynamics, responsible for significant deviation of $C(p,t)$ from its initial value N

$$\lim_{t\to 0} C(0,t) = 0 \neq \lim_{p\to 0} C(p,0)\ .$$

Free fermions

$$C_0(p,t) = N\exp\left\{-\mathrm{i}t\left(\frac{m^2}{2p}+\lambda p\right)\right\}$$

Short-time behavior for large momenta the same in the interacting and in the free theory. Put $\lambda = 0$

$$C(p,-\mathrm{i}\tau) = NK(p/\tau)\ .$$

$$q\frac{\mathrm{d}K(q)}{\mathrm{d}q} = \frac{m^2-1}{2q}K(q) - \frac{1}{2}\fint_0^\infty \mathrm{d}q'\frac{1}{q'-q}\frac{\mathrm{d}K(q')}{\mathrm{d}q'}$$

Asymptotics of $K(q)$ determined by the initial condition

$$\lim_{q\to\infty} K(q) = 1\ .$$

Infrared behavior of $K(q)$ follows

$$K(q) \sim q^{\beta_0}$$

β_n (solutions of the 't Hooft boundary condition)

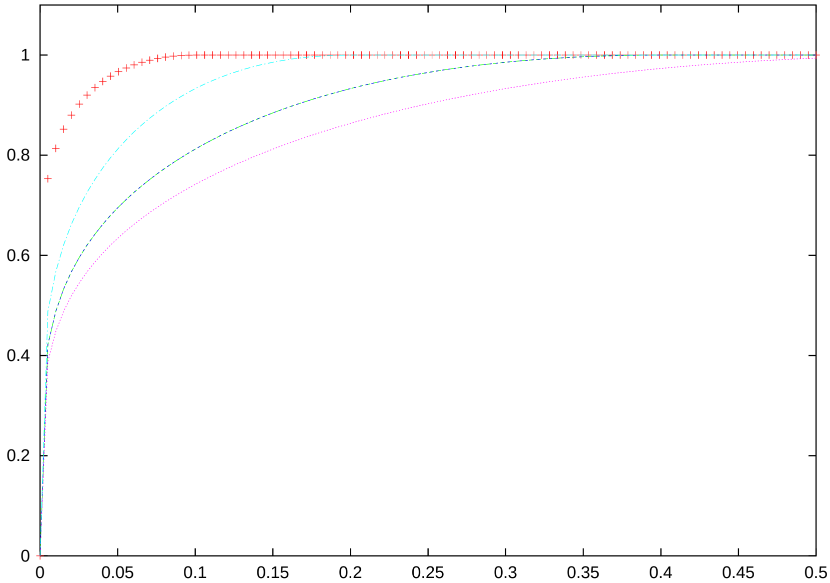
$$\pi q \cot \pi q = 1 - m^2$$

Approximate solution close to chiral limit $\beta_0 \approx \sqrt{3}m/\pi \to 0$
Confining interaction in the 't Hooft model changes essential singularity of

$$K_0(q) = \mathrm{e}^{-m^2/2q}$$

to a branch point. Consequence of presence of the gauge string in combination with infrared singularities in correlation function $C(p,t)$. In Gross-Neveu or NJL model with mass generation responsible for chiral symmetry breakdown, essential singularity preserved with renormalized value of the mass.

$$C(p,-\mathrm{i}\tau) \approx N\,(p/\tau)^{\beta_0 \theta(1-(p/\tau))}$$



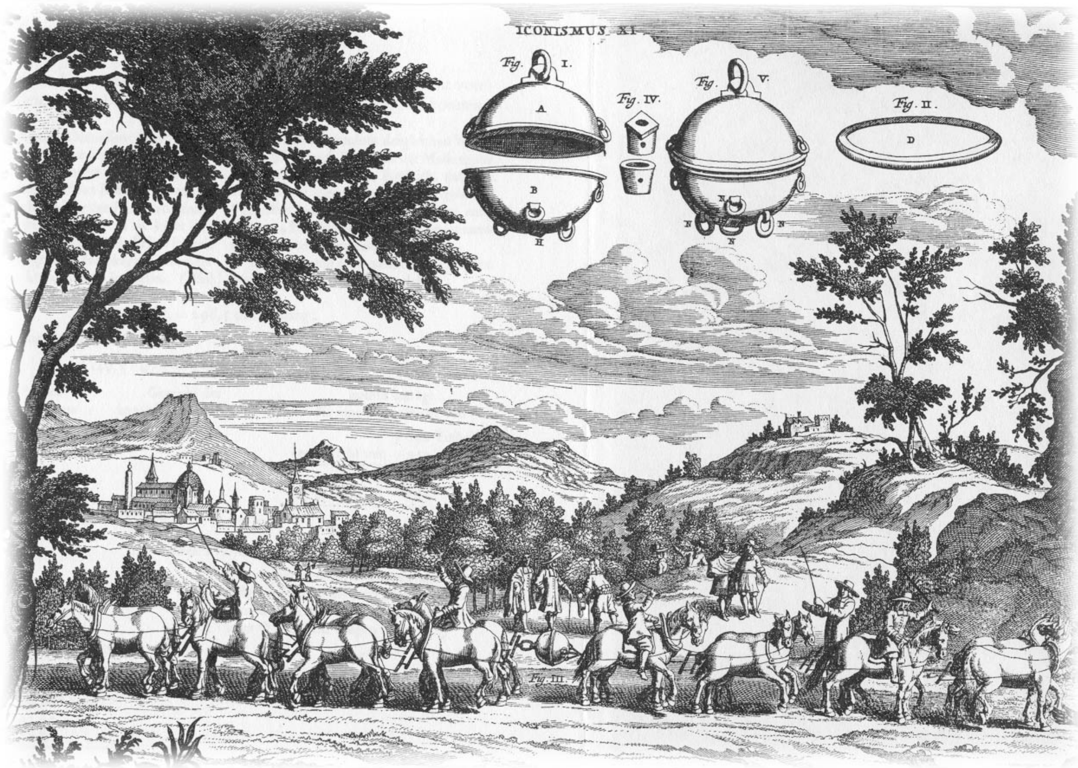
$C(p,-i\tau)$ as function of p
$\beta_0 = 0.1, \tau = 0.1, 0.2, 0.4, 0.6$

Subtle $p,\tau \to 0$ limit. Exact value of condensate in chiral limit

$$\langle\bar{\psi}\psi\rangle_z = \int \frac{\mathrm{d}p}{2\pi}\frac{m}{p}(C(p,-i\tau) - C_0(p,-i\tau)) = -\frac{N}{\sqrt{12}}$$

(Condensate can be determined also for finite m via Mellin transform of propagator .) Condensate determined by change in the infrared singularity of quark propagator. Non-perturbative short-time behavior of $C(p,t)$.

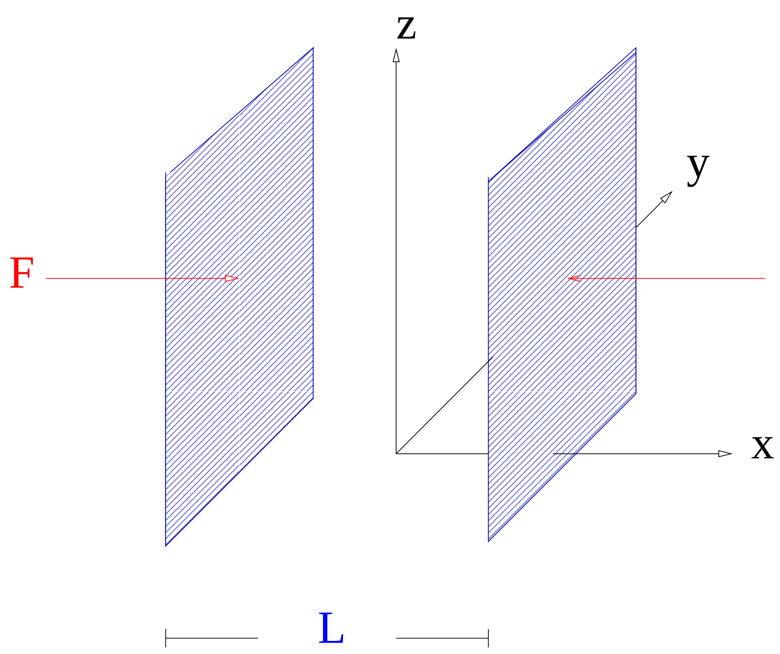
Casimir Effect *



Early Investigation of the Vacuum

O. v. Guericke demonstrates the power of the vacuum to the "Reichstag" including the emperor Ferdinand III in Regensburg 1653

* D.Steinbacher



The Vacuum of a Scalar Field on the Light-cone

2 distinct geometries

Transverse Casimir effect $x = x^1, z = x^-$

$$\varphi(x^+, x^-, x^1 + L, x^2) = \varphi(x^+, x^-, x^1, x^2)$$

Longitudinal Casimir effect $x = x^-, z = x^1$

$$\varphi(x^+, x^- + L, x^1, x^2) = \varphi(x^+, x^-, x^1, x^2)$$

$$\langle \mathcal{H} \rangle = \frac{1}{2L} \int \frac{d^3\mathbf{k}_\perp}{(2\pi)^2} \sum_n \delta\left(k_{||} - \frac{2\pi n}{L}\right) \rho(\mathbf{k})\omega(\mathbf{k})$$

$$\omega(\mathbf{k})\rho(\mathbf{k}) = \begin{matrix} |\mathbf{k}|\; e^{-\lambda^0|\mathbf{k}|} \\ \\ \frac{\mathbf{k}_\perp^2}{2k_-}\,\theta(k_-)\; e^{-\lambda^+\frac{\mathbf{k}_\perp^2}{2k_-}-\lambda^- k_-} \end{matrix}$$

Periodic boundary conditions, heat-kernel regularization of sum over single particle energies

Physical Observable - Pressure exerted on the walls

$$P = -\frac{\partial \langle L\mathcal{H} \rangle}{\partial L}$$

Standard Coordinates e.g. $k_{\parallel} = k_1$

$$\langle \mathcal{H} \rangle = \frac{3}{2\pi^2 \lambda^{0\,4}} - \frac{\pi^2}{90 L^4}$$

Transverse Casimir effect on l.c. $k_{\parallel} = k_1$

$$\langle \mathcal{H} \rangle = \frac{1}{8\pi^2 (\lambda^+ \lambda^-)^2} - \frac{\pi^2}{90 L^4}$$

Longitudinal Casimir effect on l.c. $k_{\parallel} = k_-$

$$\langle \mathcal{H} \rangle = \frac{1}{8\pi^2 (\lambda^+ \lambda^-)^2} - \frac{1}{24 \lambda^{+\,2} L^2} + \frac{\pi^2}{120 L^4} \left(\frac{\lambda^-}{\lambda^+} \right)^2$$

Standard value of pressure in transverse Casimir effect

$$P = -\frac{\pi^2}{30 L^4},$$

and ill defined in longitudinal Casimir effect.

Casimir effect observed in moving system

$$Z_\Lambda[J] = \int_{\mathrm{bc}_\Lambda} D[\varphi]\, e^{i\int d^4x\, \mathcal{L}(\varphi(x)) + i\int d^4x J\cdot\varphi}$$

Lorentz-transformed boundary conditions

$$\mathrm{bc}_\Lambda : \quad \varphi(\Lambda^{-1}(x+\ell)) = \varphi(\Lambda^{-1}(x)).$$

4-vector ℓ specifies orientation and extension of compact direction e.g.

$$\ell = \begin{array}{ll} (0, L, 0, 0) & \text{transverse} \\ \frac{1}{\sqrt{2}}(L, 0, 0, -L) & \text{longitudinal} \end{array}$$

Non-interacting massless field

$$\mathcal{L}[\varphi] = \frac{1}{2}\partial_\mu\varphi\,\partial^\mu\varphi$$

Energy-Momentum Tensor

$$\begin{aligned} \langle J_{\mu\nu}\rangle &= -(\partial^x_\mu\partial^y_\nu - \frac{1}{2}g_{\mu\nu}\partial^x_\rho\partial^{y\,\rho})\frac{\delta^2 Z_\Lambda}{\delta J(x)\delta J(y)}\Big|_{J=0,\, x\to y} \\ &= -i(\partial_\mu\partial_\nu - \frac{1}{2}g_{\mu\nu}\Box)D(\Lambda z)\Big|_{z\to 0} \end{aligned}$$

expressed by two-point function periodic in direction of ℓ

$$D(z) = \sum_n \int \frac{d^4k}{(2\pi)^3}\, e^{ikz}\, \frac{1}{k^2 + i\epsilon}\,\delta\left(k\ell - 2\pi n\right).$$

Spacelike orientation

$$L^2 = -\ell^2 > 0, \quad z^{\|} = \frac{z\ell}{L}, \quad z^{\perp} = \sqrt{z^2 + z^{\| \, 2} - i\epsilon}$$

$$D(z) = \frac{-1}{4\pi L z^{\perp}} \left[1 + \frac{1}{e^{2i\pi/L(z^{\perp}+z^{\|})} - 1} + \frac{1}{e^{2i\pi/L(z^{\perp}-z^{\|})} - 1} \right]$$

Covariance

$$D(z) = D(z^2, \ell^2, z\ell\,).$$

Limit of vanishing regulator $z/L \to 0$

$$D(z) \approx \frac{i}{4\pi} \left[\frac{1}{\pi z^2} - \frac{\pi}{3L^2} - \frac{\pi^3}{45L^4} \left(z^2 + 4\, z^{\| \, 2} \right) \right].$$

Singular, frame (Λ) and size independent (L) term does not contribute to observables

$$\langle J_{\mu\nu} \rangle = -\frac{\pi^2}{90 L^4} \Lambda^{\rho}_{\mu} \Lambda^{\sigma}_{\nu} \left[g_{\rho\sigma} - 4 \frac{\ell_\rho \, \ell_\sigma}{\ell^2} \right] \; .$$

Transverse Casimir effect $\Lambda = 1\,, l^{\mu} = \delta_{\mu,1}$

$$\langle J_{00} \rangle = \langle J_{+-} \rangle = \frac{1}{2} \langle J^{\,00} - J^{\,33} \rangle = -\frac{\pi^2}{90\, L^4}.$$

Infinite momentum frame

$$\ell^\mu = L\delta_{\mu 3}, \quad \Lambda^3_\mu = \gamma(\delta_{\mu 3} - \beta\delta_{\mu 0}), \quad \beta^2 \to 1$$

Specify order of limits

a)

$$\lim_{\beta^2 \to 1} \lim_{z \to 0} \langle J_{\mu\nu} \rangle$$

Argument of two point function

$$|(\Lambda z)^\mu|/L \to 0$$

Infinitely increasing energy density and momentum flux in 3-direction

$$\langle \mathcal{H} \rangle = -\frac{\pi^2}{90\, L^4} \left[1 + 4\beta^2\gamma^2\right], \quad \langle J_{33} \rangle = -\frac{\pi}{90 L^4}\gamma^2(3 + \beta^2)\,,$$

and finite light-cone energy density

$$\langle J_{+-} \rangle = \frac{1}{2}\left(J^{\,00} - J^{\,33}\right) = \frac{\pi^2}{90\, L^4}\,.$$

Infinite momentum frame

b)

$$\lim_{z\to 0}\lim_{\beta^2\to 1}\langle J_{\mu\nu}\rangle$$

$$\Lambda(z^{\perp}+z^{3}) \qquad \xrightarrow[\beta\to -1]{} \frac{2}{\sqrt{1+\beta}}\, z^{+} \to \infty$$

$$\Lambda(z^{\perp}-z^{3}) \qquad \xrightarrow[\beta\to -1]{} \frac{\sqrt{1+\beta}\, z^{2}}{2z^{+}} \to 0.$$

In infinite momentum frame limit

$$\tilde{L}^2 = \frac{z^{+}L}{\sqrt{1+\beta}} \gg L^2 \quad \beta \to -1$$

Lorentz-contracted length much smaller than regulator

$$\gamma^{-1}L \ll z^{+}$$

Longitudinal Casimir effect as infinite momentum frame limit

$$\langle \mathcal{H}_{lc}\rangle = \langle J_{+-}\rangle = -\frac{i}{2}\boldsymbol{\nabla}_{\perp}^{2}\, D(\Lambda z)$$

$$\approx \; -\frac{1}{2\pi^2 z^4}\left(1-\frac{4z^{+}z^{-}}{z^2}\right)+\frac{1}{24\tilde{L}^4}+\ldots.$$

Space and Lightlike Compactifications

In longitudinal Casimir effect compact direction specified by lightlike vector (l.c. coordinates)

$$\ell = (0, L, 0, 0)\,, \quad \ell^2 = 0$$

Two-point function

$$D_{lc} = D_{lc}(z^2, z\ell),$$

cannot contain a finite and regulator independent term. No meaningful definition of the energy density if compact direction coincides with x^- direction.

Compact direction can be arbitrary close to the x^- direction. Impose boundary conditions

$$\varphi(x^+, x^- + L, x^1 + sL, x^2) = \varphi(x^+, x^-, x^1, x^2).$$

and approach x^- direction ($s \to 0$) from orientations characterized by spacelike vectors

$$\ell = (0, L, sL, 0)\,, \quad \ell^2 = -s^2 L^2.$$

Lorentz-transformation consisting of rotation around the x_2 - axis by an angle $\alpha = \arcsin(1 + 2s^2)^{-1/2}$ followed by boost along the x^1 direction with $\beta = \sin\alpha$ transforms this boundary condition into transverse boundary condition

$$\varphi(x^+, x^-, x^1 + sL, x^2) = \varphi(x^+, x^-, x^1, x^2).$$

$$\langle J_{+-} \rangle = -\frac{\pi^2}{90(sL)^4}.$$

Meaningful Casimir energy for arbitrary choices of the compact spacelike direction excluding a region around the x^- direction with opening angle determined by the ratio of the components of the regulator z over the proper length sL.

Despite need for regularization the ultraviolet quantum fluctuations, Casimir effect is a long-wave length phenomenon

$$\langle \mathcal{H} \rangle \sim \begin{array}{ll} L^{-4} & m = 0 \\ e^{-mL} & m \neq 0 \end{array}$$

Contributions with small n important, (cf. sensitivity to boundary condition - repulsion for antiperiodic b.c.), i.e. discretization of physical relevance . Peculiar infrared properties of the spectrum in light-cone quantization. Divergence of the one particle energies at small light-cone momenta is regularized by z^+. Lightlike nature of compact direction in combination with sensitivity of the observables to the infinite and long wavelength properties makes energy-momentum tensor in the longitudinal Casimir effect ill-defined. Failure not caused by l.c.quantization but by lightlike compactification. Problems in DLCQ or transverse lattice approach for calculation of vacuum properties ?

Thermodynamics on Light-cone

Straightforward generalization of standard procedure to define partition function by compactifying light-cone time faced with the difficulties encountered in longitudinal Casimir effect. The light-cone time direction is lightlike
Compactification along the x^+ direction at inverse temperature β defines 4-vector with light-cone coordinates

$$\ell = (\beta, 0, 0, 0), \quad \ell^2 = 0.$$

With

$$D(z) = D(z^2, z^- \beta)$$

finite regulator independent thermodynamic quantities cannot be extracted from $D(z)$.

Equivalence between relativistic field theories at finite temperature (β) and finite extension(L) .

$$Z\left(\beta, L\right) = Z\left(L, \beta\right), \quad \epsilon\left(\beta, L\right) = -P\left(L, \beta\right) \ .$$

Energy momentum tensor at finite temperature is trivially computed once the energy momentum tensor at finite extension is known. With spacelike compact direction these elements can be evaluated on the light-cone. Connection between Casimir effect and blackbody radiation.

Phase transitions on light-cone induced by variation of extension of compact spacelike direction. In QCD phase transition, when compressing transverse extension below 1.3 fm.