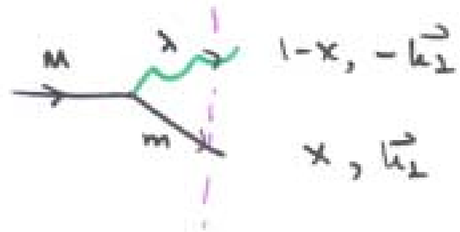
Testing ground for light-cone wavefunctions

Quantum fluctuations of electron in QED



$+ \cdots$

$L_z = -1$ $$\psi^{\uparrow}_{+1/2,+1}(x,\vec{k}_\perp) = -\sqrt{2}\,\frac{(-k^1 + i k^2)}{x(1-x)}\,\varphi(x,\vec{k}_\perp)$$

$L_z = 1$ $$\psi^{\uparrow}_{+1/2,-1}(x,\vec{k}_\perp) = -\sqrt{2}\,\frac{(k^1 + i k^2)}{1-x}\,\varphi(x,\vec{k}_\perp)$$

$L_z = 0$ $$\psi^{\uparrow}_{-1/2,+1}(x,\vec{k}_\perp) = -\sqrt{2}\left(M - \frac{m}{x}\right)\varphi(x,\vec{k}_\perp)$$

$$\varphi(x,\vec{k}_\perp) = \frac{e}{\sqrt{1-x}}\,\frac{1}{M^2 - \frac{k_\perp^2 + m^2}{x} - \frac{k_\perp^2 + \lambda^2}{1-x}}$$

S. Drell + SJB

Huang, Ma, Schmidt, SJB

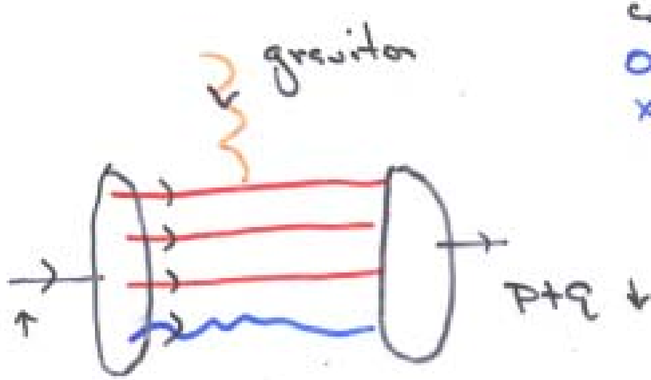
Theorem

Anomalous gravitomagnetic moment $\underline{B(0) = 0}$

manifestation of correspondence principle

Okun & Kobzarev (62)

X. Ji, Teryaev

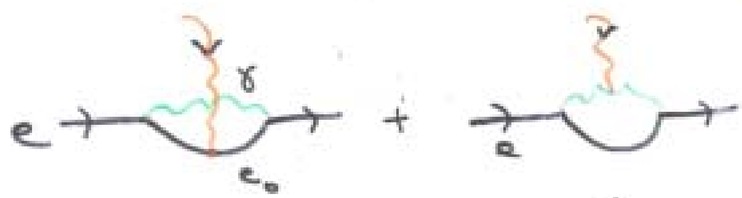


$$\sum_{i=1}^{n} B_i(0) = 0 .$$

$$\langle p+q \downarrow \,|\, \frac{T^{++}(0)}{2P^{+2}} \,|\, p \uparrow \rangle = (q_1 - i q_2)\, B(q^2)$$

e.g. quantum fluctuations of electron

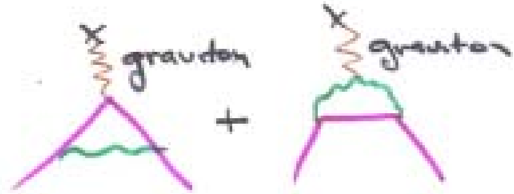
Hwang, Ma, Schmidt SJB



$\frac{\alpha}{3\pi}$ $\qquad$ $-\frac{\alpha}{3\pi}$

$B_e + B_\gamma = 0$ at $q^2 = 0$.

Anomalous Gravitomagnetic Moment $B(0)$

QED  

$\boxed{F_2(0) = \frac{\alpha}{2\pi}}$

$\boxed{B(0) = \frac{\alpha}{3\pi} - \frac{\alpha}{3\pi} = 0.}$

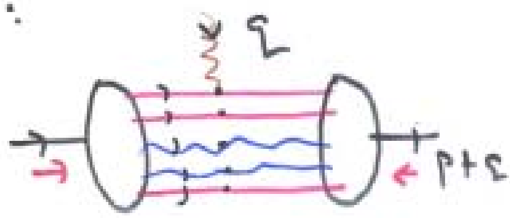
$B(q^2) \sim \frac{\alpha}{\pi} \sqrt{q^2/m^2}$

$\boxed{\text{Equivalence Principle : } B(0) = 0}$

any spin 1/2 system

Okun + Kobzarev (62)
X. Ji , Teryaev

L.F. :

$\lim_{q^2 \to 0} \sum_{i=1}^{n}$ 

$(\Delta L_z = 1)$

$\sum_{i=1}^{n} B_i(0) = 0$
Fock state by Fock state.

Result of Lorentz prop of LF wavefunctions.

Hwang, Ma, Schmidt, SJB

$\boxed{\text{Key Question for LGTh} \quad B_p(0) = 0 \ ?}$

Important indicator of lattice errors.

The total contribution from the fermion and boson constituents is

$$B(q^2) = B_{\rm f}(q^2) + B_{\rm b}(q^2) \tag{1}$$

$$\begin{aligned}
&= 2Mg^2 \int \frac{{\rm d}^2\vec{k}_\perp {\rm d}x}{16\pi^3} \frac{(m+xM)}{x} \\
&\times \Big\{ \frac{1}{[M^2 - ((\vec{k}_\perp + (1-x)\vec{q}_\perp)^2 + m^2)/x - ((\vec{k}_\perp + (1-x)\vec{q}_\perp)^2 + \lambda^2)/(1-x)]} \\
&\quad - \frac{1}{[M^2 - ((\vec{k}_\perp - x\vec{q}_\perp)^2 + m^2)/x - ((\vec{k}_\perp - x\vec{q}_\perp)^2 + \lambda^2)/(1-x)]} \Big\} \\
&\times \quad \frac{1}{[M^2 - (\vec{k}_\perp^2 + m^2)/x - (\vec{k}_\perp^2 + \lambda^2)/(1-x)]}
\end{aligned} \tag{2}$$

$$= \frac{Mg^2}{8\pi^2} \int_0^1 d\alpha \int_0^1 dx \ (1-x)(m+xM) \tag{3}$$

$$\times \Big(\frac{1}{\alpha(1-\alpha)\frac{1-x}{x}\vec{q}_\perp^2 - M^2 + \frac{m^2}{x} + \frac{\lambda^2}{1-x}} - \frac{1}{\alpha(1-\alpha)\frac{x}{1-x}\vec{q}_\perp^2 - M^2 + \frac{m^2}{x} + \frac{\lambda^2}{1-x}} \Big) .$$

At zero momentum transfer

$$B(0) = B_{\rm f}(0) + B_{\rm b}(0) = 0 , \tag{4}$$

which is an example of the vanishing of the anomalous gravitomagnetic moment.

It is important to identify the $n-1$ independent relative momenta of the n-particle Fock state.

$$
\begin{aligned}
-\frac{B(0)}{2M} &= \lim_{q^1_\perp \to 0} \frac{\partial}{\partial q^1_\perp} \left\langle P+q,\uparrow \left| \frac{T^{++}(0)}{2(P^+)^2} \right| P,\downarrow \right\rangle \\
&= \lim_{q^1_\perp \to 0} \frac{\partial}{\partial q^1_\perp} \left\langle \Psi^\uparrow(P^+=1,\vec{P}_\perp=\vec{q}_\perp)) \left| \frac{T^{++}(0)}{2(P^+)^2} \right| \Psi^\downarrow(P^+=1,\vec{P}_\perp=\vec{0}_\perp) \right\rangle \\
&= \lim_{q^1_\perp \to 0} \frac{\partial}{\partial q^1_\perp} \sum_a \int \prod_{k=1}^{n-1} \frac{\mathrm{d}^2\vec{k}_{\perp k}\mathrm{d}x_k}{16\pi^3} \psi_a^{\uparrow *}\big(x_1,x_2,\cdots,x_{n-1},(1-x_1-x_2-\cdots-x_{n-1}), \\
&\qquad \vec{k}'_{\perp 1},\vec{k}'_{\perp 2},\cdots,\vec{k}'_{\perp n-1},(-\vec{k}'_{\perp 1}-\vec{k}'_{\perp 2}-\cdots-\vec{k}'_{\perp n-1})\big) \\
&\times \Big[\sum_{i=1}^{n-1} x_i + (1-x_1-x_2-\cdots-x_{n-1})\Big] \\
&\times \psi_a^{\downarrow}\big(x_1,x_2,\cdots,x_{n-1},(1-x_1-x_2-\cdots-x_{n-1}), \\
&\qquad \vec{k}_{\perp 1},\vec{k}_{\perp 2},\cdots,\vec{k}_{\perp n-1},(-\vec{k}_{\perp 1}-\vec{k}_{\perp 2}-\cdots-\vec{k}_{\perp n-1})\big)\ .
\end{aligned}
$$

Using integration by parts,

$$
\begin{aligned}
&-\frac{B_a(0)}{2M} = \\
&= \int \prod_{k=1}^{n-1} \frac{\mathrm{d}^2\vec{k}_{\perp k}\mathrm{d}x_k}{16\pi^3} \psi_a^{\uparrow *}\big(x_1,x_2,\cdots,x_{n-1},(1-x_1-x_2-\cdots-x_{n-1}), \\
&\qquad \vec{k}_{\perp 1},\vec{k}_{\perp 2},\cdots,\vec{k}_{\perp n-1},(-\vec{k}_{\perp 1}-\vec{k}_{\perp 2}-\cdots-\vec{k}_{\perp n-1})\big) \\
&\times \Big[\sum_{i=1}^{n-1} x_i\big((-1+x_i)\frac{\partial}{\partial k^1_{\perp i}} + \sum_{j\neq i}^{n-1} x_j \frac{\partial}{\partial k^1_{\perp j}}\big) + (1-x_1-x_2-\cdots-x_{n-1}) \sum_{j=1}^{n-1} x_j \frac{\partial}{\partial k^1_{\perp j}}\Big] \\
&\times \psi_a^{\downarrow}\big(x_1,x_2,\cdots,x_{n-1},(1-x_1-x_2-\cdots-x_{n-1}), \\
&\qquad \vec{k}_{\perp 1},\vec{k}_{\perp 2},\cdots,\vec{k}_{\perp n-1},(-\vec{k}_{\perp 1}-\vec{k}_{\perp 2}-\cdots-\vec{k}_{\perp n-1})\big) \\
&= \int \prod_{k=1}^{n-1} \frac{\mathrm{d}^2\vec{k}_{\perp k}\mathrm{d}x_k}{16\pi^3} \psi_a^{\uparrow *}\big(x_1,x_2,\cdots,x_{n-1},(1-x_1-x_2-\cdots-x_{n-1}), \\
&\qquad \vec{k}_{\perp 1},\vec{k}_{\perp 2},\cdots,\vec{k}_{\perp n-1},(-\vec{k}_{\perp 1}-\vec{k}_{\perp 2}-\cdots-\vec{k}_{\perp n-1})\big) \\
&\times \Big[\sum_{i=1}^{n-1} \big(-1+\sum_{j=1}^{n-1} x_j + (1-x_1-x_2-\cdots-x_{n-1})\big) x_i \frac{\partial}{\partial k^1_{\perp i}}\Big] \\
&\times \psi_a^{\downarrow}\big(x_1,x_2,\cdots,x_{n-1},(1-x_1-x_2-\cdots-x_{n-1}), \\
&\qquad \vec{k}_{\perp 1},\vec{k}_{\perp 2},\cdots,\vec{k}_{\perp n-1},(-\vec{k}_{\perp 1}-\vec{k}_{\perp 2}-\cdots-\vec{k}_{\perp n-1})\big) \\
&= 0\ .
\end{aligned}
$$

Thus the contribution $B_a(0)$ from each contributing Fock state a to the total anomalous gravitomagnetic moment $B(0)$ vanishes separately.

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+ Dae Sung Hwang

Exact Formulae For Exclusive Semi-Leptonic B-decay

e.g. $B^- \to l^- \bar{\nu} D^0$

$B^- \to l^- \bar{\nu} \pi^0$

$$\mathcal{M} = \mathcal{M}^\mu L_\mu$$

Example of off-diagonal matrix element

$$\mathcal{M}^\mu = \langle B | J^\mu(0) | A \rangle$$

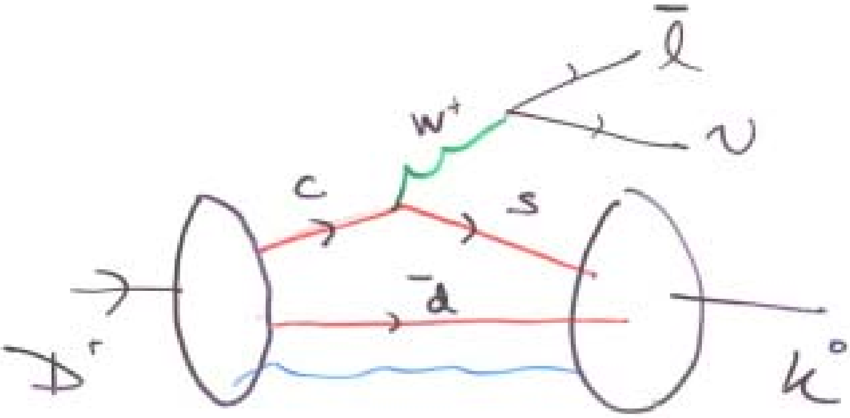
$$J^\mu(0) = j^\mu(0)$$

interaction picture

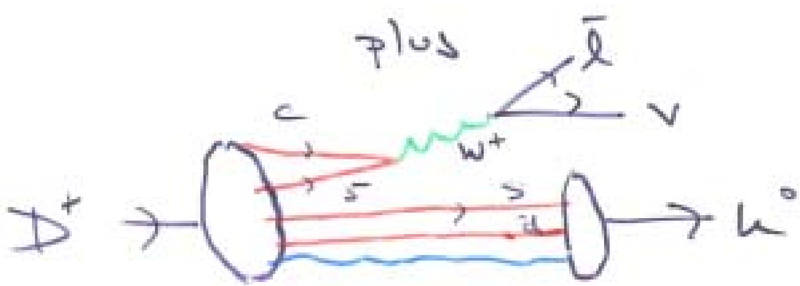
Light-Cone Wavefunctions:

B, D

exclusive decays



plus



New Phenomena in weak decays

Compton Sum Rules

X. Ji:
Radyushkin
S&D + D.S. Hwang
M. Diehl
Burkardt

$$\int_0^1 dx \; H(x,t,\zeta) = F_1(t)$$

ζ- indep.
quark by quark

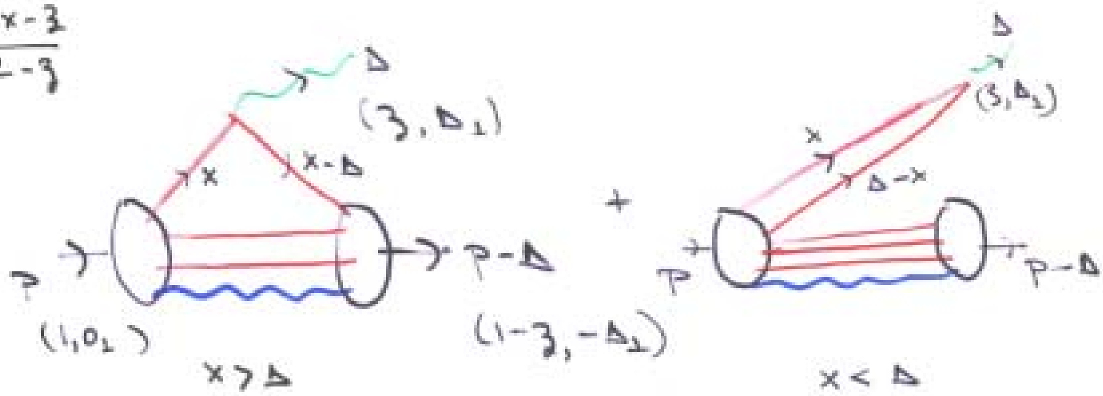
$$\int_0^1 dx \; E(x,t,\zeta) = F_2(t)$$

$$\int_0^1 dx \; \tilde{x} \, H(x,t,\zeta) = A_q(t)$$

gravitational form factors

$$\int_0^1 dx \; \tilde{x} \, E(x,t,\zeta) = B_q(t)$$

$$\tilde{x} = \frac{2x-\zeta}{2-\zeta}$$

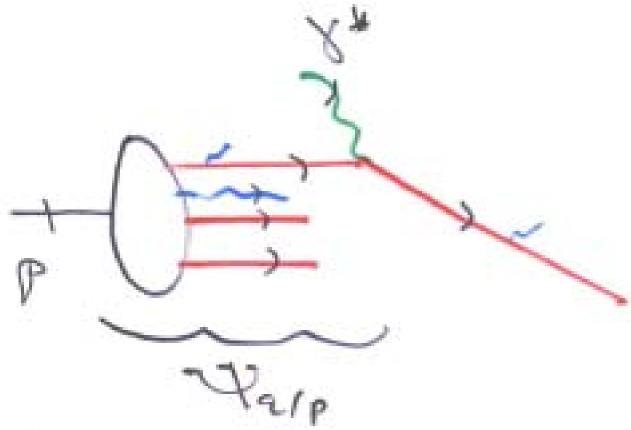


ζ- indep at fixed

$$t = \Delta^2 = -\left(\frac{\zeta^2 M^2 + \Delta_\perp^2}{1-\zeta}\right) \quad !$$

Feynman
Bjorken-Paschos

Foundation of Parton Model



proved in
Yukawa theory
Drell, Levy, Yan

DGLAP evol.

$$F_2(x, Q^2) = \sum_{q \in p} e_q^2 \, q(x, Q^2)$$

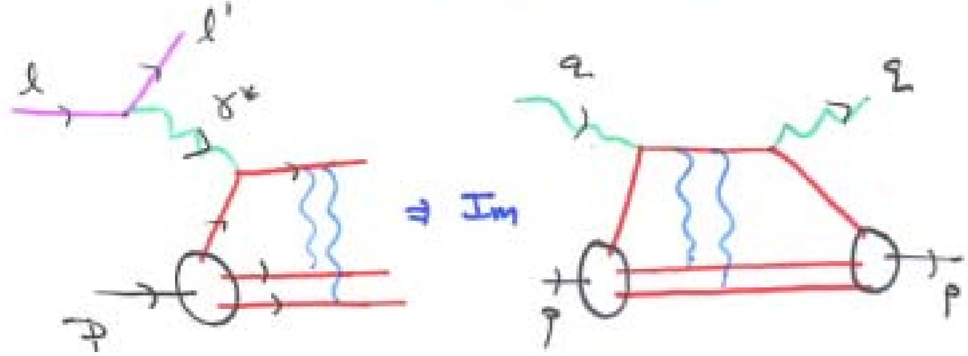
$q(x, Q^2)$ determined by target
wavefunctions $\Psi_{q/p}$

All interactions after photon acts
inconsequential !

→ phase or power-law suppressed

BHMPS ⇒ QCD not parton model + DGLAP

New perspectives on Final-State-Interactions



$$f_{q/N}(x_{Bj}, Q^2) = \frac{1}{8\pi}\int dy^- \, e^{-ix_B p^+ y^-}$$

$$\langle N(p)|\bar{q}(y^-)\gamma^+ P \, e^{ig\int_0^{y^-} dw^- A^+(w^-)} q(0)|N(p)\rangle$$

Usual argument $A^+ = 0$ gauge

no effect from FSI !

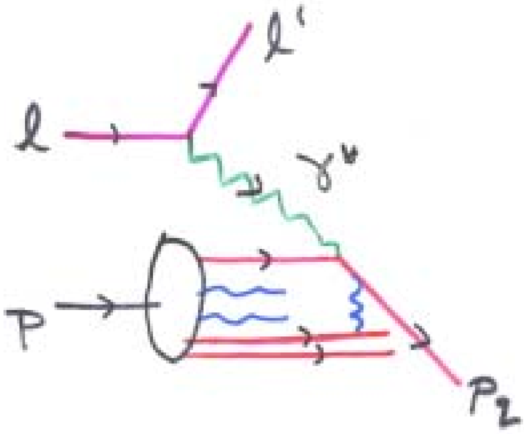
phase irrelevant

$\therefore$ Identify $f_{q/N}$ with LC Prob. Dist

$$P_{q/N}(x_B, Q^2) = \sum_n \int^{k_{\perp i}^2 < Q^2} [\Pi \, dx \, d^2k_\perp] |\psi_n(x, \vec{k}_{\perp i})|^2$$

$$\sum_j \delta(x_B - x_j)$$

Unexpected Role of Final State Interactions in Deep Inelastic Scattering



gluon exchange after photon acts

not in LF WF

* Single-spin asymmetry $\vec{S}_p \cdot \vec{q} \times \vec{p}_q$
Bjorken-scaling
* Diffraction at Leading Twist
* Nuclear Shadowing (interference of diff channels)
* Energy Loss, p_T Broadening

Diffraction, Nuclear Shadowing, Pomeron not in nuclear wavefunction!

References

"Structure functions are not parton probabilities"

S.J.B., P. Hoyer, N. Marchal, S. Peigne + F. Sannino

PRD 65 114025 (2002) hep-ph/0104291

"Final-state interactions and Single-spin Asymmetries in Semi-inclusive Deep Inelastic Scatt."

S.J.B., Dae Sung Hwang, I. Schmidt

PLB 530 99 (2002) hep-ph/0201296

* J.C. Collins hep-ph/0204004

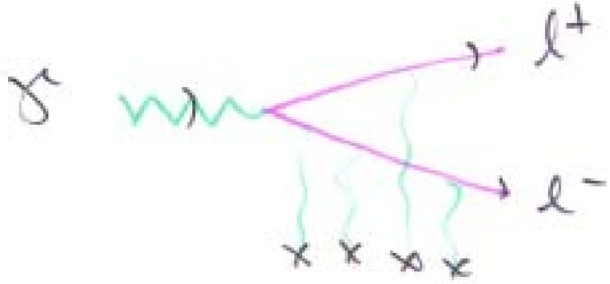
* X. Ji + F. Yuan hep-ph/0206057

* S.J.B, A. Hebecker + E. Quack PRD 55, 2584 (1997)

Bethe - Heitler Pair Production

– all orders in Coulomb Field

Bethe - Maximon - Davies PR 93, 768, 788 (1954)



– $$\sigma = \frac{28}{9}\frac{\alpha(Z\alpha)^2}{m_\ell^2}\left[\log\frac{2\nu}{m_\ell} - \frac{109}{42} - f(Z\alpha)\right]$$

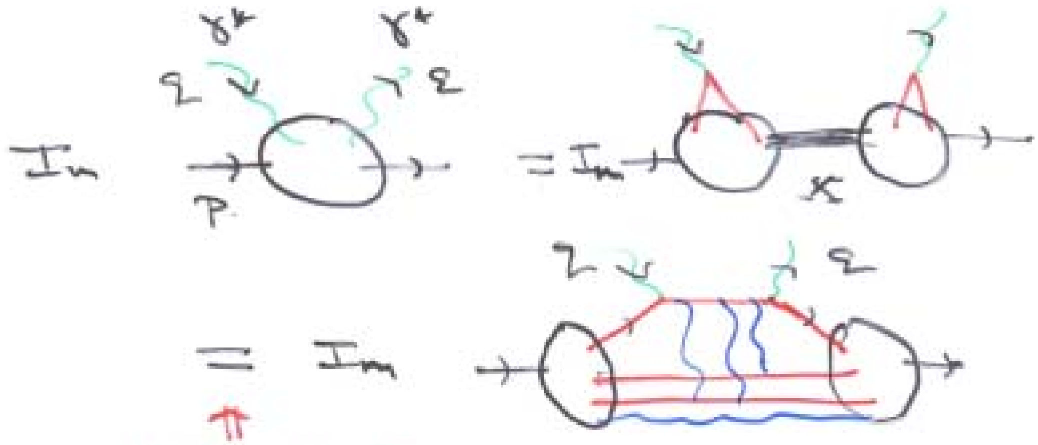
– $$f(Z\alpha) = (Z\alpha)^2\sum_{n=1}^{\infty}\frac{1}{n(n^2+(Z\alpha)^2)}\,.$$

Unitary (Eikonal) approx. fails.

$\log\nu$ only in Born approx.

Usual proof:

QCD Factorization for virtual compton amplitude:

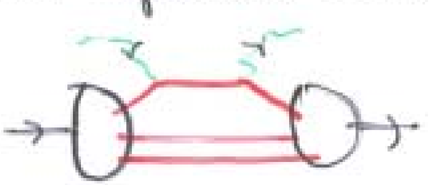


leading twist

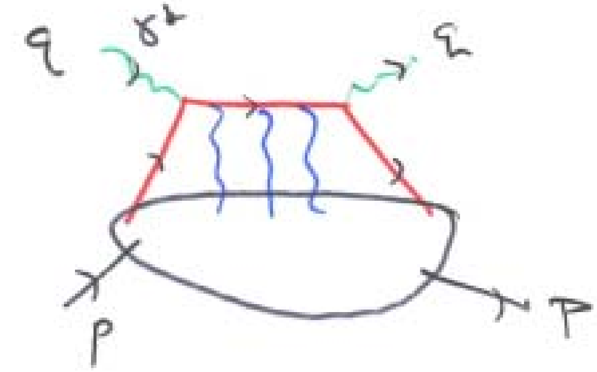
$$P_{q/N}(x_B, Q^2) = \frac{1}{8\pi}\int dy^- \, e^{-i x_B p^+ y^-}$$

$$\langle N(p)|\, \bar{q}(y^-)\, \gamma^+ \, \mathcal{P}\, e^{ig\int_0^{y^-} dw^- A^+(w^-)}\, q(0)|N(p)\rangle$$

Choose light-cone gauge: $A^+ = 0$,
Path-ordered exponential $= 1$

$\Rightarrow$ Im  $= \sum_n |\psi_n|^2$

Usual argument



$q^+ \leq 0$
frame

$$F_{q/N}(x_B, Q^2) = \frac{1}{8\pi}\int dy^-\, e^{-i x_B p^+ y^-}$$

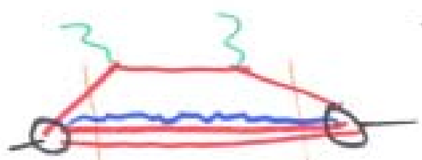
$$\langle p|\,\bar{q}(y^-)\,\gamma^+\, P\, e^{ig\int_0^{y^-} dw^- A^+(w^-)}\, q(0)\,|p\rangle$$

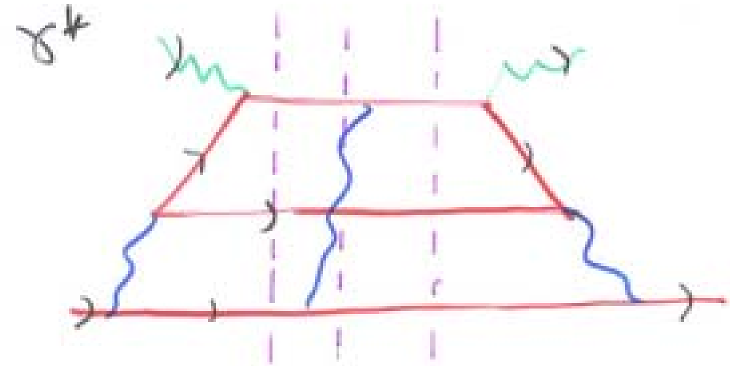
Choose $A^+ = 0$ gauge

No rescattering of struck quark

$$\Rightarrow \quad F_{q/N}(x_B, Q^2) = q(x_B, Q^2)$$

$$= \sum_n |\psi_n|^2$$





Usual argument: no time for f.s.i.

- three denominators of order ν
- numerator coupling finite in l.c.g.

$\therefore$ f.s.i. suppressed by power of $\frac{1}{\nu}$

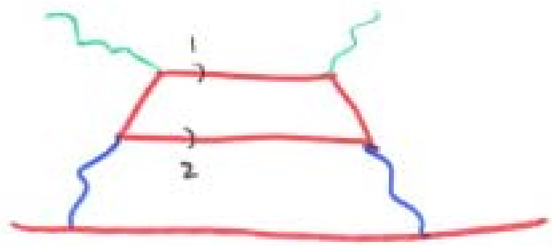
Equivalent to setting P.O.E. = 1.

$$\int_0^{y^-} dw^- \, A^+(w^-) = 0 .$$

in $A^+ = 0$ gauge.

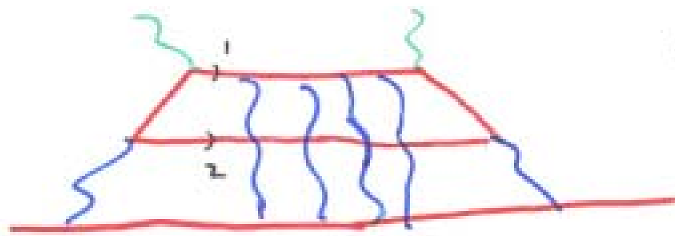
But argument is wrong!

Consider



$$q^+ \simeq 0$$

In Feynman gauge, must keep



Shadowing effect from on-shell states

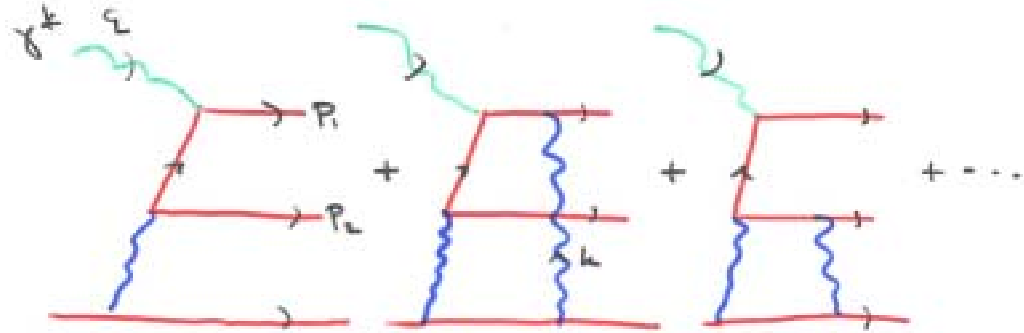
$$|M|^2 \Rightarrow \left| \frac{\sin[g^2 W(r_\perp, R_\perp)/2]}{g^2 W(r_\perp, R_\perp)/2} \, M_0 \right|^2$$

$$W(r_\perp, R_\perp) = \int d^2k_\perp \, \frac{1 - e^{+i r_\perp \cdot \vec{k}_\perp}}{k_\perp^2} \, e^{i \vec{R}_\perp \cdot \vec{k}_\perp}$$
$$= \frac{1}{2\pi} \log\left(\frac{|\vec{R}_\perp + \vec{r}_\perp|}{|R_\perp|}\right)$$

Explicit calculation of FSI

BHMPS

$q^+ \leq 0$ frame



LFTOPT: gluon exchanged after photon acts

Find non-zero leading-twist FSI effect

Leading twist diffraction, Eikonal form

Shadowing corrections, not Coulomb phase

Gauge-independent:

Checked: Feynman, LCG $[A^+=0]$ $\left\{ \begin{array}{l} ML \\ PV \\ k \end{array} \right.$ prescriptions

Lesson for LCG:

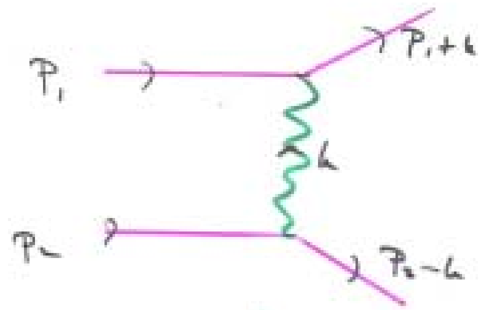
$$g^{\mu\nu} - \frac{n^\mu k^\nu + k^\nu n^\mu}{n \cdot k}$$

At pole: $n \cdot k = k^+ = O(\frac{1}{\nu})$.

Light-cone gauge evaluation

$n^\mu = (0, 2, 0_\perp)$

$n^2 = 0$



$2p_1 \cdot k + k^2 = 0$

$$\mathcal{M} = e^2 (2p_1 + k)_\mu \frac{\left[-g^{\mu\nu} + \frac{n^\mu k^\nu + k^\mu n^\nu}{n \cdot k} \right]}{k^2 + i\epsilon} (2p_2 - k)_\nu$$

where: $n \cdot k = k^+ = \frac{-k_\perp^2}{p_1^-} \to 0$ for $s \gg -t$

Dominant term in l.c.g. from non-eikonal current

$$\mathcal{M} = e^2 \frac{-k_\perp^2 \frac{2p_2^+}{k^+}}{k^2 + i\epsilon} \Rightarrow -2e^2 \frac{s}{t}$$

$\vec{A}_\perp$ plays crucial role in lcg: $k^+ = O\left(\frac{1}{p_1^-}\right)$

Similarly: $\int dw^\mu A_\mu \neq \int dw^- A^+$ in l.c.g.