Physical Interpretation for the Generalized Parton Distributions $H(x, 0, -\Delta_\perp^2)$ and $E(x, 0, -\Delta_\perp^2)$

or: What DVCS has to do with the distribution of partons in the transverse plane

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Outline

- Motivation
- Deep-inelastic scattering (DIS)
- Generalized parton distributions (GPDs)
- Probabilistic interpretation of GPDs
  - $H(x, 0, -\Delta_\perp^2) \longrightarrow q(x, \mathbf{b}_\perp)$
  - $\tilde{H}(x, 0, -\Delta_\perp^2) \longrightarrow \Delta q(x, \mathbf{b}_\perp)$
  - $E(x, 0, -\Delta_\perp^2) \longrightarrow \perp$ displacement of quark distributions
- Summary

Motivation:

(see talk by A. Belitski)

- Interesting observation: X.Ji, PRL**78**,610(1997)

$$\langle J_q \rangle = \frac{1}{2} \int_0^1 dx\, x \left[H_q(x,0,0) + E_q(x,0,0) \right]$$

$$\boxed{\text{DVCS}} \quad \Leftrightarrow \quad \boxed{\text{GPDs}} \quad \Leftrightarrow \quad \boxed{\vec{J}_q}$$

$\hookrightarrow$ lots of works about GPDs

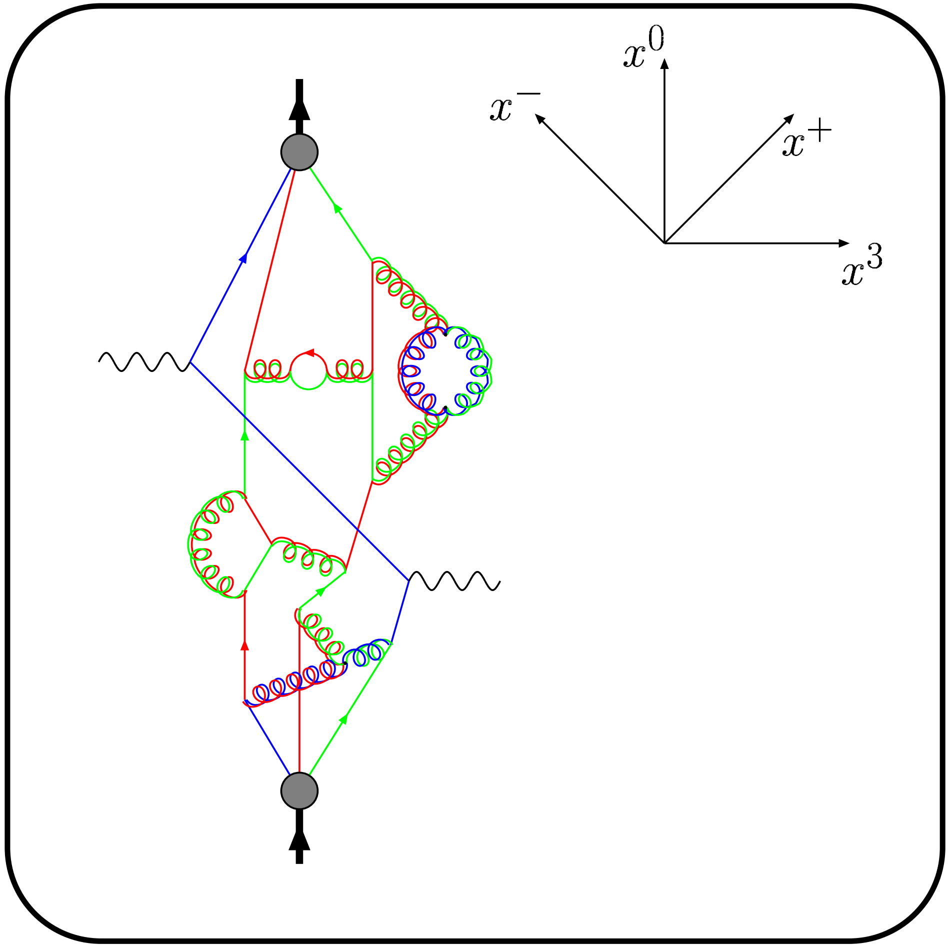
$\hookrightarrow$ GPDs very useful tool linking variety of observables

- But: what other “physical information” about the nucleon can we obtain by measuring/calculating GPDs?

DIS

light-cone coordinates:

$$x^+ = \left(x^0 + x^3\right)/\sqrt{2}$$
$$x^- = \left(x^0 - x^3\right)/\sqrt{2}$$

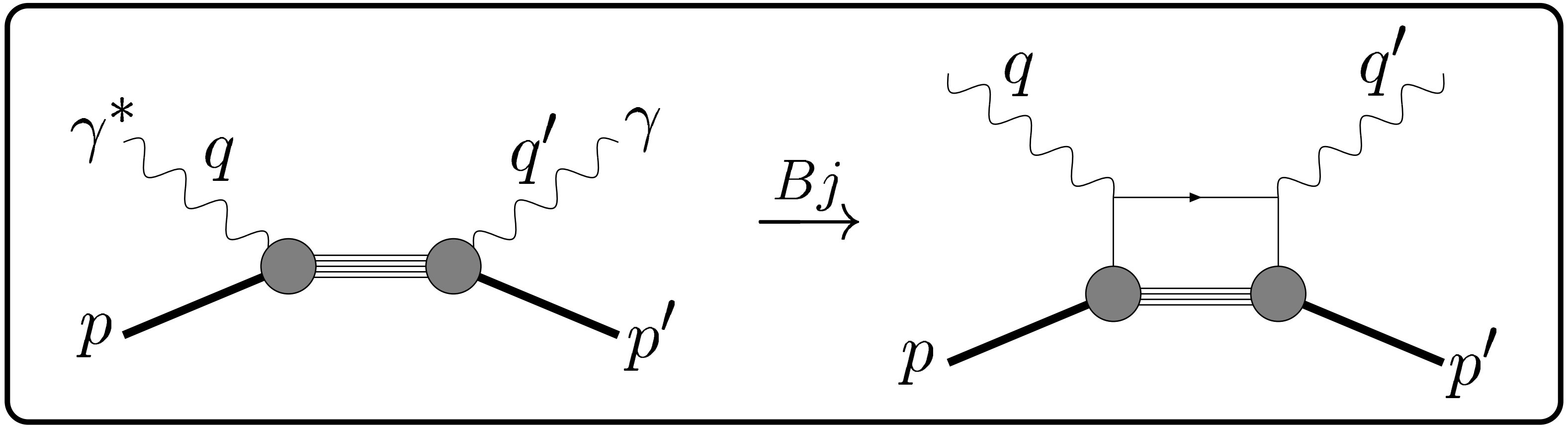


DIS related to correlations along light–cone

$$q(x_{Bj}) = \int \frac{dx^-}{2\pi} \langle P|\overline{q}(0^-, \mathbf{0}_\perp)\gamma^+ q(x^-, \mathbf{0}_\perp)|P\rangle \, e^{ix^- x_{Bj} P^+}$$

No information about transverse position of partons!

Deeply Virtual Compton Scattering (DVCS)



$$T^{\mu\nu} = i\int d^4z\, e^{i\bar{q}\cdot z}\left\langle p' \left| T J^{\mu}\left(-\frac{z}{2}\right) J^{\nu}\left(\frac{z}{2}\right)\right| p\right\rangle$$

$$\overset{Bj}{\hookrightarrow} \quad \frac{g_{\perp}^{\mu\nu}}{2}\int_{-1}^{1} dx \left(\frac{1}{x-\xi+i\varepsilon}+\frac{1}{x+\xi-i\varepsilon}\right) H(x,\xi,\Delta^2)\bar{u}(p')\gamma^{+}u(p) \quad + \quad \ldots$$

$$\bar{q} = (q+q')/2 \qquad \Delta = p'-p \qquad x_{Bj} \equiv -q^2/2p\cdot q = 2\xi(1+\xi)$$

Generalized Parton Distributions (GPDs)

$$\int \frac{dx^-}{2\pi} e^{ix^-\bar{p}^+x} \left\langle p' \middle| \bar{q}\left(-\frac{x^-}{2}\right)\gamma^+ q\left(\frac{x^-}{2}\right) \middle| p \right\rangle = H(x,\xi,\Delta^2)\bar{u}(p')\gamma^+ u(p) + E(x,\xi,\Delta^2)\bar{u}(p')\frac{i\sigma^{+\nu}\Delta_\nu}{2M}u(p)$$

$$\int \frac{dx^-}{2\pi} e^{ix^-\bar{p}^+x} \left\langle p' \middle| \bar{q}\left(-\frac{x^-}{2}\right)\gamma^+\gamma_5 q\left(\frac{x^-}{2}\right) \middle| p \right\rangle = \tilde{H}(x,\xi,\Delta^2)\bar{u}(p')\gamma^+\gamma_5 u(p) + \tilde{E}(x,\xi,\Delta^2)\bar{u}(p')\frac{\gamma_5\Delta^+}{2M}u(p)$$

back

Parton Interpretation

- x is mean long. momentum fraction carried by active quark
- $\Delta^+ = 2\xi\bar{p}^+$, i.e. ξ is the long. momentum transfer on quark/target
- $\Delta_\perp$ is perp. momentum transfer on target
- In general no probabilistic interpretation since initial and final state not the same
- instead: interpretation as transition amplitude
- GPDs tell how much quarks with momentum fraction x (in IMF) contribute to form factor
- depend on 3 variables since 1.) x is 'measured'; 2.) LC correlation singles out z-direction

What is Physics of GPDs ?

- Definition of GPDs resembles that of form factors

$$\left\langle p' \left| \hat{O} \right| p \right\rangle = H(x,\xi,\Delta^2)\bar{u}(p')\gamma^+ u(p) + E(x,\xi,\Delta^2)\bar{u}(p')\frac{i\sigma^{+\nu}\Delta_\nu}{2M}u(p)$$

with $\hat{O} \equiv \int \frac{dx^-}{2\pi} e^{ix^-\bar{p}^+x}\bar{q}\left(-\frac{x^-}{2}\right)\gamma^+ q\left(\frac{x^-}{2}\right)$

$\hookrightarrow$ relation between PDFs and GPDs similar to relation between a charge and a form factor

$\hookrightarrow$ If form factors can be interpreted as Fourier transforms of charge distributions in position space, what is the analogous physical interpretation for GPDs ?

Form Factors vs. GPDs

operator	forward matrix elem.	off-forward matrix elem.	position space
$\bar{q}\gamma^+ q$	Q	$F(t)$	$\rho(\vec{r})$
$\int \frac{dx^- e^{ixp^+x^-}}{4\pi} \bar{q}\left(\frac{-x^-}{2}\right)\gamma^+ q\left(\frac{x^-}{2}\right)$	$q(x)$	$H(x,\xi,t)$	?

Form Factors vs. GPDs

operator	forward matrix elem.	off-forward matrix elem.	position space
$\bar{q}\gamma^+ q$	Q	$F(t)$	$\rho(\vec{r})$
$\int \frac{dx^- e^{ixp^+x^-}}{4\pi} \bar{q}\left(\frac{-x^-}{2}\right)\gamma^+ q\left(\frac{x^-}{2}\right)$	$q(x)$	$H(x,0,t)$	$q(x,\mathbf{b}_\perp)$

$q(x,\mathbf{b}_\perp)$ = impact parameter dependent PDF

Impact parameter dependent PDF

- define state that is localized in $\perp$ position:

$$\left|p^+, \mathbf{R}_\perp = \mathbf{0}_\perp, \lambda\right\rangle \equiv \mathcal{N} \int d^2\mathbf{p}_\perp \left|p^+, \mathbf{p}_\perp, \lambda\right\rangle$$

Note: $\perp$ boosts in IMF form Galilean subgroup
$\Rightarrow$ this state has $\mathbf{R}_\perp \equiv \sum_i x_i \mathbf{b}_{\perp,i} = \mathbf{0}_\perp$
(cf.: working in CM frame in nonrel. physics)

- define impact parameter dependent PDF

$$q(x, \mathbf{b}_\perp) \equiv \int \frac{dx^-}{4\pi} \langle p^+, \mathbf{0}_\perp | \bar{\psi}(-\frac{x^-}{2}, \mathbf{b}_\perp) \gamma^+ \psi(\frac{x^-}{2}, \mathbf{b}_\perp) | p^+, \mathbf{0}_\perp \rangle \, e^{ixp^+x^-}$$

- use translational invariance to relate to same matrix element that appears in def. of GPDs

$$q(x,\mathbf{b}_\perp) \equiv \int dx^- \langle p^+, \mathbf{R}_\perp = \mathbf{0}_\perp | \bar{\psi}(-\frac{x^-}{2}, \mathbf{b}_\perp)\gamma^+\psi(\frac{x^-}{2}, \mathbf{b}_\perp) | p^+, \mathbf{R}_\perp = \mathbf{0}_\perp \rangle e^{ixp^+x^-}$$

$$= |\mathcal{N}|^2 \int d^2\mathbf{p}_\perp \int d^2\mathbf{p}'_\perp \int dx^- \langle p^+, \mathbf{p}'_\perp | \bar{\psi}(-\frac{x^-}{2}, \mathbf{b}_\perp)\gamma^+\psi(\frac{x^-}{2}, \mathbf{b}_\perp) | p^+, \mathbf{p}_\perp \rangle e^{ixp^+x^-}$$

- use translational invariance to relate to same matrix element that appears in def. of GPDs

$$q(x,\mathbf{b}_\perp) \equiv \int dx^- \left\langle p^+,\mathbf{R}_\perp=\mathbf{0}_\perp\right|\bar{\psi}(-\frac{x^-}{2},\mathbf{b}_\perp)\gamma^+\psi(\frac{x^-}{2},\mathbf{b}_\perp)\left|p^+,\mathbf{R}_\perp=\mathbf{0}_\perp\right\rangle e^{ixp^+x^-}$$

$$= |\mathcal{N}|^2\int d^2\mathbf{p}_\perp\int d^2\mathbf{p}'_\perp\int dx^-\left\langle p^+,\mathbf{p}'_\perp\right|\bar{\psi}(-\frac{x^-}{2},\mathbf{b}_\perp)\gamma^+\psi(\frac{x^-}{2},\mathbf{b}_\perp)\left|p^+,\mathbf{p}_\perp\right\rangle e^{ixp^+x^-}$$

$$= |\mathcal{N}|^2\int d^2\mathbf{p}_\perp\int d^2\mathbf{p}'_\perp\int dx^-\left\langle p^+,\mathbf{p}'_\perp\right|\bar{\psi}(-\frac{x^-}{2},\mathbf{0}_\perp)\gamma^+\psi(\frac{x^-}{2},\mathbf{0}_\perp)\left|p^+,\mathbf{p}_\perp\right\rangle e^{ixp^+x^-}$$

$$\times e^{i\mathbf{b}_\perp\cdot(\mathbf{p}_\perp-\mathbf{p}'_\perp)}$$

- use translational invariance to relate to same matrix element that appears in def. of GPDs

$$q(x,\mathbf{b}_\perp) \equiv \int dx^- \left\langle p^+,\mathbf{R}_\perp=\mathbf{0}_\perp\right| \bar{\psi}(-\frac{x^-}{2},\mathbf{b}_\perp)\gamma^+\psi(\frac{x^-}{2},\mathbf{b}_\perp)\left|p^+,\mathbf{R}_\perp=\mathbf{0}_\perp\right\rangle e^{ixp^+x^-}$$

$$= |\mathcal{N}|^2\int d^2\mathbf{p}_\perp \int d^2\mathbf{p}'_\perp \int dx^- \left\langle p^+,\mathbf{p}'_\perp\right| \bar{\psi}(-\frac{x^-}{2},\mathbf{b}_\perp)\gamma^+\psi(\frac{x^-}{2},\mathbf{b}_\perp)\left|p^+,\mathbf{p}_\perp\right\rangle e^{ixp^+x^-}$$

$$= |\mathcal{N}|^2\int d^2\mathbf{p}_\perp \int d^2\mathbf{p}'_\perp \int dx^- \left\langle p^+,\mathbf{p}'_\perp\right| \bar{\psi}(-\frac{x^-}{2},\mathbf{0}_\perp)\gamma^+\psi(\frac{x^-}{2},\mathbf{0}_\perp)\left|p^+,\mathbf{p}_\perp\right\rangle e^{ixp^+x^-} \times e^{i\mathbf{b}_\perp\cdot(\mathbf{p}_\perp-\mathbf{p}'_\perp)}$$

$$= |\mathcal{N}|^2\int d^2\mathbf{p}_\perp \int d^2\mathbf{p}'_\perp H\left(x,0,-\left(\mathbf{p}'_\perp-\mathbf{p}_\perp\right)^2\right) e^{i\mathbf{b}_\perp\cdot(\mathbf{p}_\perp-\mathbf{p}'_\perp)}$$

$$\hookrightarrow \boxed{q(x,\mathbf{b}_\perp) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} H(x,-\Delta_\perp^2) e^{-i\mathbf{b}_\perp\cdot\Delta_\perp}}$$

- $q(x, \mathbf{b}_\perp)$ has physical interpretation of a density

$$\boxed{\begin{array}{c} q(x, \mathbf{b}_\perp) \geq 0 \quad \text{for} \quad x > 0 \\ q(x, \mathbf{b}_\perp) \leq 0 \quad \text{for} \quad x < 0 \end{array}}$$

Discussion

- GPDs allow simultaneous determination of longitudinal momentum and transverse position of partons

$$q(x,\mathbf{b}_\perp) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} H(x,0,-\Delta_\perp^2) e^{-i\mathbf{b}_\perp\cdot\Delta_\perp}$$

- $q(x,\mathbf{b}_\perp)$ has interpretation as density (positivity constraints!)

$$\begin{aligned} q(x,\mathbf{b}_\perp) &\sim \left\langle p^+,\mathbf{0}_\perp \left| b^\dagger(xp^+,\mathbf{b}_\perp) b(xp^+,\mathbf{b}_\perp) \right| p^+,\mathbf{0}_\perp \right\rangle \\ &= \left| b(xp^+,\mathbf{b}_\perp) |p^+,\mathbf{0}_\perp\rangle \right|^2 \geq 0 \end{aligned}$$

$\hookrightarrow$ positivity constraint on models

- Nonrelativistically such a result not surprising! Absence of relativistic corrections to identification $H(x,0,-\Delta_\perp^2) \xrightarrow{FT} q(x,\mathbf{b}_\perp)$ due to Galilean subgroup in IMF
- $\mathbf{b}_\perp$ distribution measured w.r.t. $\mathbf{R}_\perp^{CM} \equiv \sum_i x_i \mathbf{r}_{i,\perp}$
$\hookrightarrow$ width of the $\mathbf{b}_\perp$ distribution should go to zero as $x \to 1$, since the active quark becomes the $\perp$ center of momentum in that limit!
$\hookrightarrow$ $H(x,t)$ must become t-indep. as $x \to 1$.
- very similar results for impact parameter dependent polarized quark distributions

$$\Delta q(x,\mathbf{b}_\perp) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} \tilde{H}(x,0,-\Delta_\perp^2) e^{-i\mathbf{b}_\perp\cdot\Delta_\perp}$$

- Use intuition about nucleon structure in position space to make predictions for GPDs:
  large x: quarks from localized valence 'core',
  small x: contributions from larger ' meson cloud'
  $\hookrightarrow$ expect a gradual increase of the t-dependence ($\perp$ size) of $H(x,0,t)$ as x decreases
- small x, expect tranverse size to increase
- model: $H_q(x,0,-\Delta_\perp^2) = q(x)e^{-a\Delta_\perp^2(1-x)\ln\frac{1}{x}}$.
  (consistent with Regge behavior at small x and quark counting for $x \to 1$)

Other topics

- The physics of $E(x, 0, t)$
- QCD evolution
- extrapolating to $\xi = 0$

Summary

- DVCS allows probing GPDS

$$\int \frac{dx^-}{2\pi} e^{ixp^+x^-} \left\langle p' \left| \bar{\psi}\left(-\frac{x^-}{2}\right) \gamma^+ \psi\left(\frac{x^-}{2}\right) \right| p \right\rangle$$

- GPDs resemble both PDFs and form factors: defined through matrix elements of light-cone correlation, but $\Delta \equiv p' - p \neq 0$.
- t-dependence of GPDs at $\xi = 0$ (only $\perp$ momentum transfer) $\Rightarrow$ Fourier transform of impact parameter dependent parton distributions $q(x, \mathbf{b}_\perp)$

$\hookrightarrow$ knowledge of GPDs for $\xi = 0$ allows determining distribution of partons in the $\perp$ plane

$$q(x,\mathbf{b}_\perp) = \int \frac{d^2\mathbf{b}_\perp}{(2\pi)^2} H(x,0,-\Delta_\perp^2) e^{-i\mathbf{b}_\perp\cdot\Delta_\perp}$$
$$\Delta q(x,\mathbf{b}_\perp) = \int \frac{d^2\mathbf{b}_\perp}{(2\pi)^2} \tilde{H}(x,0,-\Delta_\perp^2) e^{-i\mathbf{b}_\perp\cdot\Delta_\perp}$$

$\hookrightarrow$ GPDs provide novel information about nonperturbative parton structure of nucleons: distribution of partons in $\perp$ plane
$L_x \sim \langle yp_z - zp_y \rangle$ only lowest $\mathbf{b}_\perp$ moment of that information

- $q(x,\mathbf{b}_\perp)$, $\Delta q(x,\mathbf{b}_\perp)$ have probabilistic interpretation, e.g. $q(x,\mathbf{b}_\perp) > 0$ for $x > 0$
- universal prediction: large x partons more localized in $\mathbf{b}_\perp$ than small x partons

- $\frac{\Delta_\perp}{2M} E(x, -\Delta_\perp^2)$ describes how the momentum distribution of unpolarized partons in the $\perp$ plane gets transversely shifted (distorted) if is nucleon polarized in $\perp$ direction..
- published in: M.B., PRD **62**, 71503 (2000), hep-ph/0105324, and hep-ph/0207047; see also ($\xi \neq 0$) M.Diehl, hep-ph/0205208.

The physics of $E(x,0,-\Delta_\perp^2)$

So far: only unpolarized (or long. polarized) nucleon

In general, use ($\Delta^+ = 0$)

$$\begin{aligned}
\int \frac{dx^-}{4\pi} e^{ip^+x^-x} \left\langle P{+}\Delta,\uparrow \right| \bar{q}(0)\,\gamma^+ q(x^-) \left| P,\uparrow \right\rangle &= H(x,0,-\Delta_\perp^2) \\
\int \frac{dx^-}{4\pi} e^{ip^+x^-x} \left\langle P{+}\Delta,\uparrow \right| \bar{q}(0)\,\gamma^+ q(x^-) \left| P,\downarrow \right\rangle &= -\frac{\Delta_x - i\Delta_y}{2M} E(x,0,-\Delta_\perp^2).
\end{aligned}$$

Consider nucleon polarized in x direction (in IMF)

$|X\rangle \equiv |p^+, \mathbf{R}_\perp = \mathbf{0}_\perp, \uparrow\rangle + |p^+, \mathbf{R}_\perp = \mathbf{0}_\perp, \downarrow\rangle.$

$\hookrightarrow$ quark distribution for this state

$$q_X(x,\mathbf{b}_\perp) = q(x,\mathbf{b}_\perp) - \frac{1}{2M}\frac{\partial}{\partial b_y}\int \frac{d^2\Delta_\perp}{(2\pi)^2} E(x,-\Delta_\perp^2) e^{-i\mathbf{b}_\perp\cdot\Delta_\perp}$$

$q_X(x,\mathbf{b}_\perp)$ gets shifted (distorted) compared to longitudinally polarized nucleons

- mean displacement ($\perp$ flavor dipole moment)

$$d_y^q \equiv \int dx \int d^2\mathbf{b}_\perp q_X(x,\mathbf{b}_\perp) b_y = \frac{1}{2M}\int dx E_q(x,0,0)$$

with

$$\kappa_{u/d} \equiv F_2^{u/d}(0) = \mathcal{O}(1) \quad \Rightarrow \quad d_y^q = \mathcal{O}(0.1 fm)$$

- CM for flavor q shifted relative to CM for whole proton by

$$\int dx \int d^2\mathbf{b}_\perp q_X(x,\mathbf{b}_\perp) x b_y = \frac{1}{2M}\int dx\, x E_q(x,0,0)$$

- $q_X(x, \mathbf{b}_\perp) \geq 0$

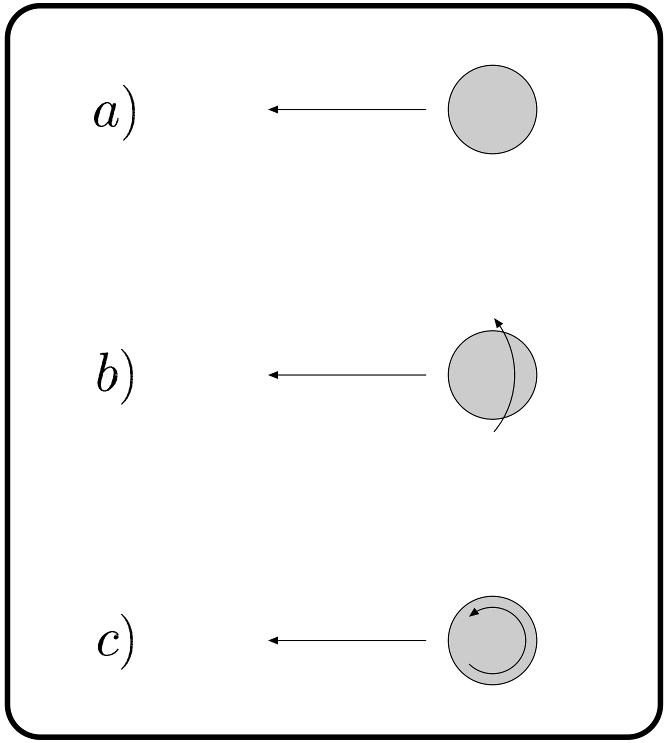
$\hookrightarrow$ positivity constraint for FT of $E(x, 0, -\Delta_\perp^2)$:

$$\left| \frac{\nabla_{b_\perp}}{2M} \int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{-i\mathbf{b}_\perp\cdot\Delta_\perp} E(x, -\Delta_\perp^2) \right| < q(x, \mathbf{b}_\perp)$$

- polarized GPD with helicity flip $\Rightarrow$

$$\left| \frac{\nabla_{b_\perp}}{2M} \int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{-i\mathbf{b}_\perp\cdot\Delta_\perp} E(x, -\Delta_\perp^2) \right|^2 < |q(x, \mathbf{b}_\perp)|^2 - |\Delta q(x, \mathbf{b}_\perp)|^2$$

physical origin for $\perp$ distortion



Comparison of a non-rotating sphere that moves in z direction with a sphere that spins at the same time around the z axis and a sphere that spins around the x axis When the sphere spins around the x axis, the rotation changes the distribution of momenta in the z direction (adds/subtracts to velocity for $y > 0$ and $y < 0$ respectively). For the nucleon the resulting modifi cation of the (unpolarized) momentum distribution is described by $E(x, t)$.

back

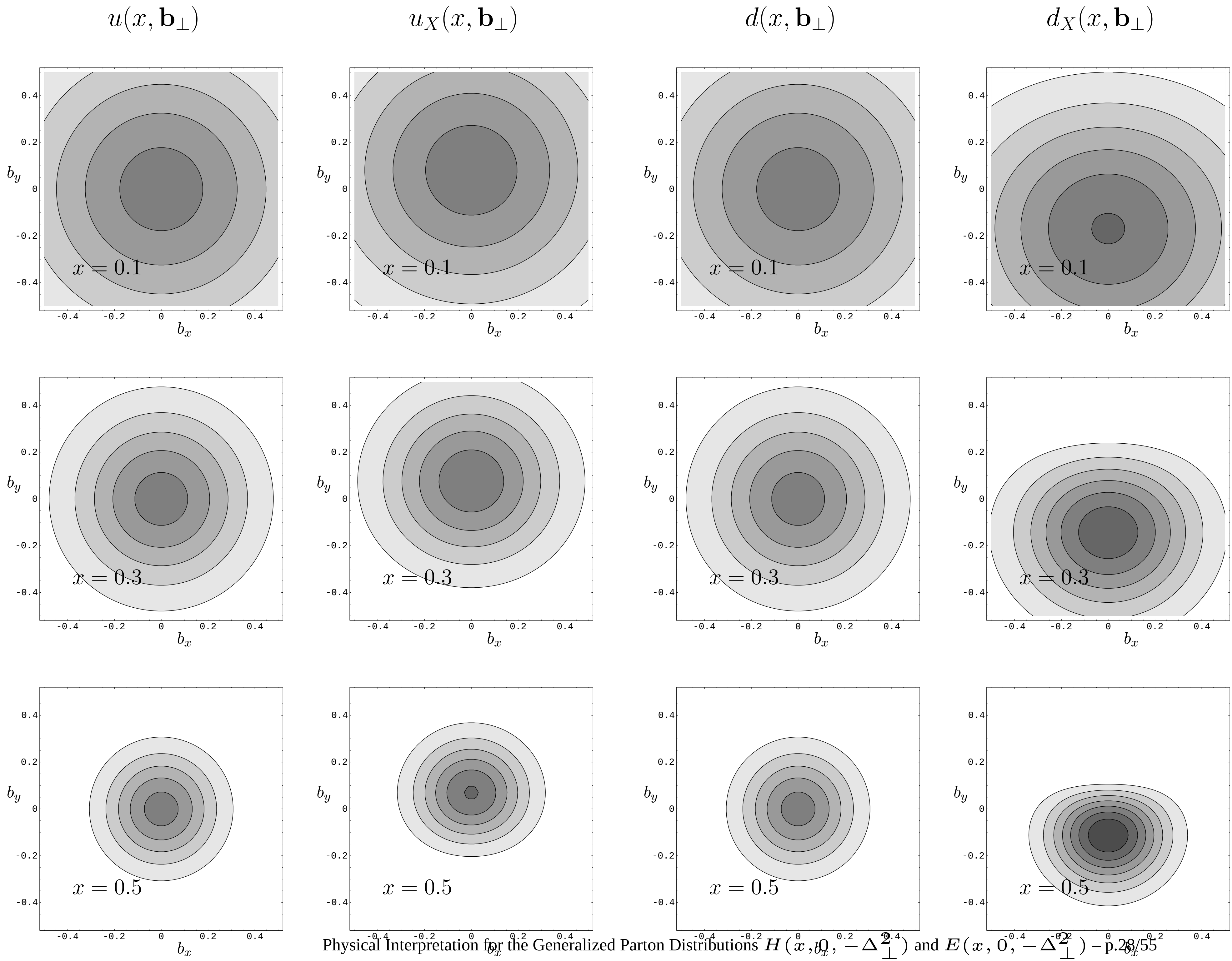
simple model for $E_q(x, 0, -\Delta_\perp^2)$

- $$\begin{aligned} E_u(x,0,-\Delta_\perp^2) &= \frac{\kappa_u}{2} H_u(x,0,-\Delta_\perp^2) \\ E_d(x,0,-\Delta_\perp^2) &= \kappa_d H_d(x,0,-\Delta_\perp^2) \end{aligned}$$

with $H_q(x,0,-\Delta_\perp^2) = q(x) e^{-a\Delta_\perp^2 (1-x) \ln\frac{1}{x}}$ and

$\kappa_u = 2\kappa_p + \kappa_n = 1.673 \qquad \kappa_d = 2\kappa_n + \kappa_p = -2.033.$

- satisfies $\int dx E_q(x,0,0) = \kappa_q$


$u(x, \mathbf{b}_\perp)$
$u_X(x, \mathbf{b}_\perp)$
$d(x, \mathbf{b}_\perp)$
$d_X(x, \mathbf{b}_\perp)$
$x = 0.1$
$x = 0.3$
$x = 0.5$
b_x
b_y

Application: $\perp$ hyperon polarization

model for hyperon polarization in $pp \to Y + X$ ($Y \in \Lambda, \Sigma, \Xi$) at high energy:

- peripheral scattering
- $s\bar{s}$ produced in overlap region, i.e. on "inside track"

$\hookrightarrow$ if Y deflected to left then s produced on left side of Y (and vice versa)

$\hookrightarrow$ if $\kappa_s > 0$ then intermediate state has better overlap with final state Y that has spin down (looking into the flight direction)

$\hookrightarrow$ remarkable prediction: $\boxed{\vec{P}_Y \sim -\kappa_s^Y \vec{p}_P \times \vec{p}_Y.}$

back

- SU(3) analysis for κ_s^B yields (assuming $|\kappa_s^p| \ll |\kappa_u^p|, |\kappa_d^p|$)

$$\begin{aligned}
\kappa_s^\Lambda &= \kappa^p + \kappa_s^p = 1.79 + \kappa_s^p \\
\kappa_s^\Sigma &= \kappa^p + 2\kappa^n + \kappa_s^p = -2.03 + \kappa_s^p \\
\kappa_s^\Xi &= 2\kappa^p + \kappa^n + \kappa_s^p = 1.67 + \kappa_s^p.
\end{aligned}$$

$\hookrightarrow$ expect (polarization $\mathcal{P}$ w.r.t. $\vec{p}_P \times \vec{P}_Y$)

$$\mathcal{P}_\Lambda < 0 \qquad \mathcal{P}_\Sigma > 0 \qquad \mathcal{P}_\Xi < 0$$

- exp. result:

$$0 < \mathcal{P}_{\Sigma^0} \approx \mathcal{P}_{\Sigma^-} \approx \mathcal{P}_{\Sigma^+} \approx -\mathcal{P}_\Lambda \approx -\mathcal{P}_{\Xi^0} \approx -\mathcal{P}_{\Xi^-}$$

back

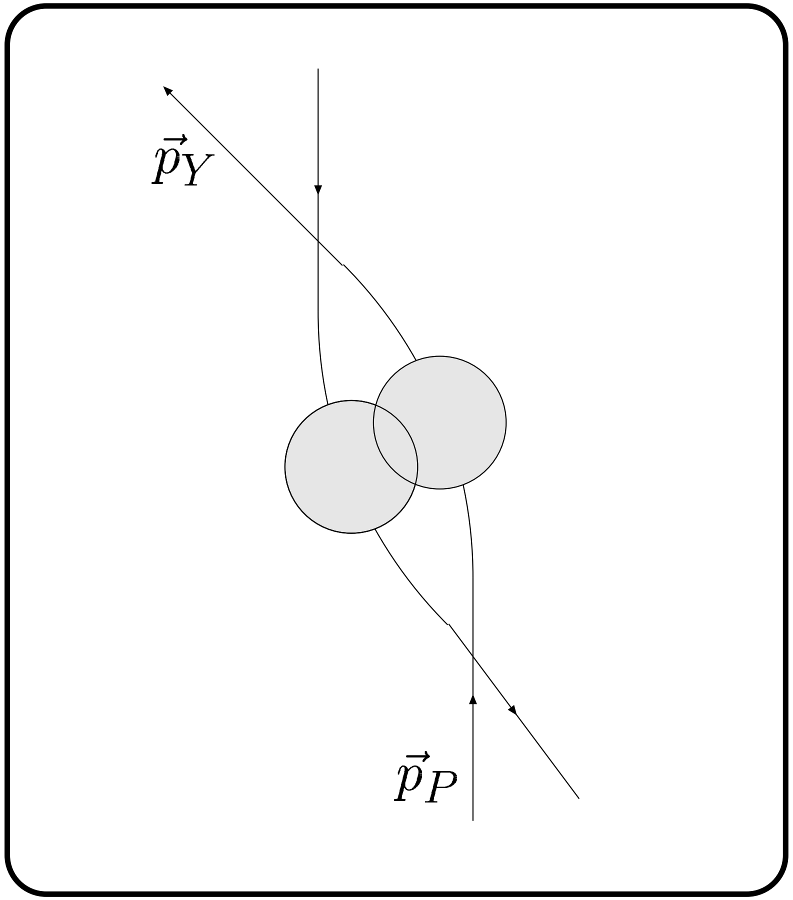


Figure 1: $P + P(\bar{P}) \longrightarrow Y + \bar{Y}$ where the incoming P (from bottom) is deflected to the left during the reaction. The $s\bar{s}$ pair is assumed to be produced roughly in the overlap region, i.e. on the left 'side' of the Y.

back

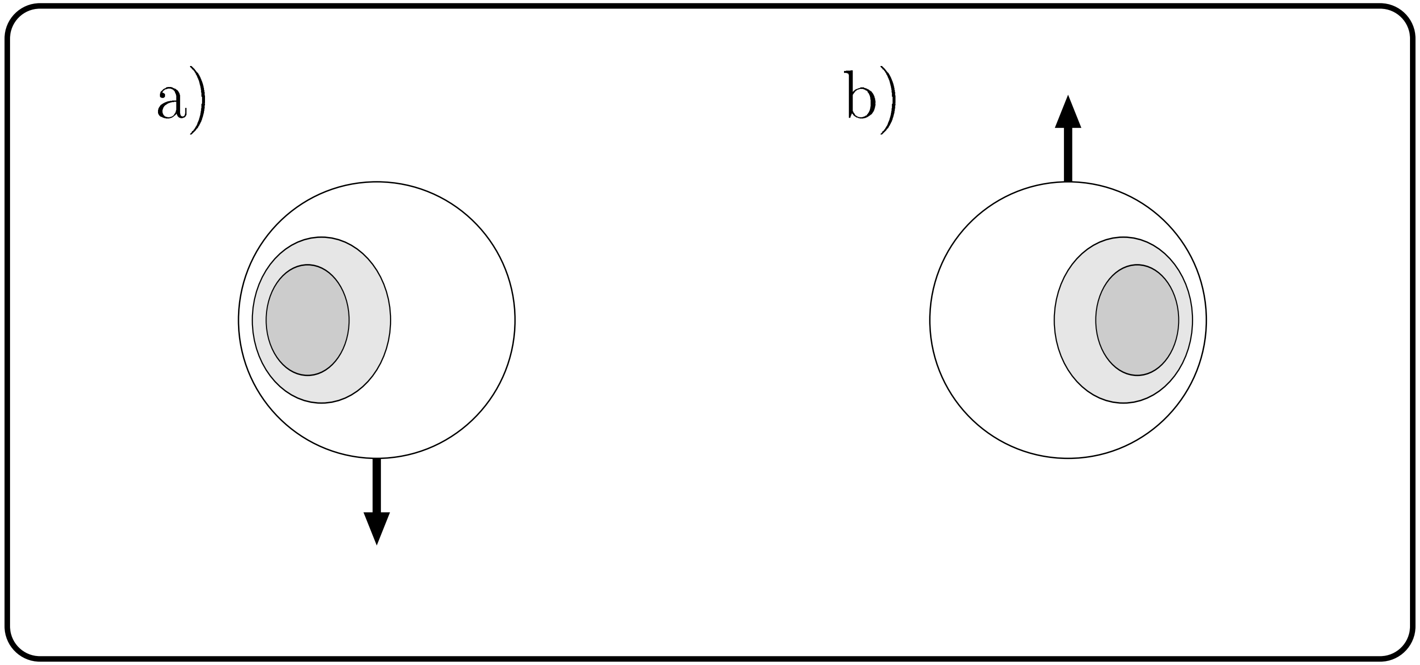


Figure 2: Schematic view of the transverse distortion of the s quark distribution (in grayscale) in the transverse plane for a transversely polarized hyperon with $\kappa_s^Y > 0$. The view is (from the rest frame) into the direction of motion (i.e. momentum into plane) for a hyperon that moves with a large momentum. In the case of spin down (a), the s-quarks get distorted towards the left, while the distortion is to the right for the case of spin up (b).

back

extrapolating to $\xi = 0$

- bad news: $\xi = 0$ not directly accessible in DVCS since long. momentum transfer necessary to convert virtual γ into real γ
- good news: moments of GPDs have simple ξ-dependence (polynomials in ξ)
$\hookrightarrow$ should be possible to extrapolate!

even moments of $H(x, \xi, t)$:

$$
\begin{aligned}
H_n(\xi, t) &\equiv \int_{-1}^{1} dx x^{n-1} H(x, \xi, t) = \sum_{i=0}^{\left[\frac{n-1}{2}\right]} A_{n,2i}(t)\xi^{2i} + C_n(t) \\
&= A_{n,0}(t) + A_{n,2}(t)\xi^2 + \ldots + A_{n,n-2}(t)\xi^{n-2} + C_n(t)\xi^n,
\end{aligned}
$$

i.e. for example

$$\int_{-1}^{1} dx x H(x,\xi,t) = A_{2,0}(t) + C_2(t)\xi^2.$$

- For n^{th} moment, need $\frac{n}{2}+1$ measurements of $H_n(\xi,t)$ for same t but different ξ to determine $A_{n,2i}(t)$.
- GPDs @ $\xi = 0$ obtained from $H_n(\xi = 0, t) = A_{n,0}(t)$
- similar prodecure exists for moments of $\tilde{H}$

back

QCD evolution

So far ignored! Can be easily included because

- For $t \ll Q^2$, leading order evolution t-independent
- For $\xi = 0$ evolution kernel for GPDs same as DGLAP evolution kernel

likewise:

- impact parameter dependent PDFs evolve such that different $\mathbf{b}_\perp$ do not mix (as long as $\perp$ spatial resolution much smaller than Q^2)

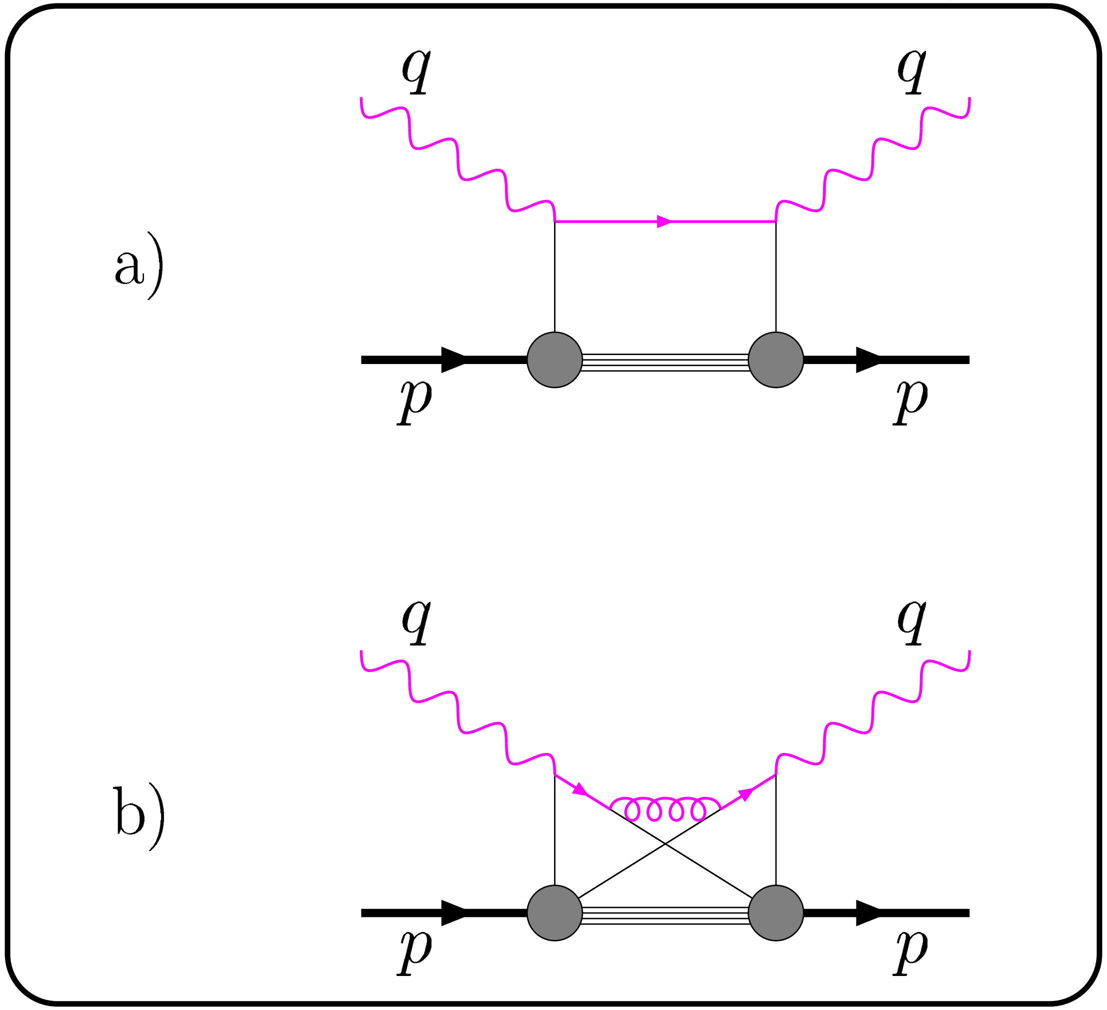
$\hookrightarrow$ above results consistent with QCD evolution:

$$\boxed{\begin{aligned} H(x,0,-\Delta_\perp^2,Q^2) &= \int d^2b_\perp q(x,\mathbf{b}_\perp,Q^2)e^{-i\mathbf{b}_\perp\mathbf{\Delta}_\perp} \\ \tilde{H}(x,0,-\Delta_\perp^2,Q^2) &= \int d^2b_\perp \Delta q(x,\mathbf{b}_\perp,Q^2)e^{-i\mathbf{b}_\perp\mathbf{\Delta}_\perp} \end{aligned}}$$

where QCD evolution of $H, \tilde{H}, q, \Delta q$ is described by DGLAP and is independent on both $\mathbf{b}_\perp$ and $\Delta_\perp^2$, provided one does not look at scales in $\mathbf{b}_\perp$ that are smaller than $1/Q$.

back

suppression of crossed diagrams



Flow of the large momentum q through typical diagrams contributing to the forward Compton amplitude. a) 'handbag' diagrams; b) 'cat's ears' diagram. Diagram b) is suppressed at large q due to the presence of additional propagators.

back

Form factor vs. charge distribution (non-rel.)

- define state that is localized in position space (center of mass frame)

$$\left|\vec{R}=\vec{0}\right\rangle \equiv \mathcal{N} \int d^3\vec{p}\,|\vec{p}\rangle$$

- define charge distribution (for this localized state)

$$\rho(\vec{r}) \equiv \left\langle \vec{R}=\vec{0}\right| j^0(\vec{r}) \left|\vec{R}=\vec{0}\right\rangle$$

- use translational invariance to relate to same matrix element that appears in def. of form factor

$$\rho(\vec{r}) \equiv \left\langle \vec{R}=\vec{0}\right| j^0(\vec{r}) \left|\vec{R}=\vec{0}\right\rangle$$

$$= |\mathcal{N}|^2 \int d^3\vec{p} \int d^3\vec{p}' \, \langle \vec{p}'| \, j^0(\vec{r}) \, |\vec{p}\rangle$$

$$= |\mathcal{N}|^2 \int d^3\vec{p} \int d^3\vec{p}' \langle \vec{p}'| \, j^0(\vec{0}) \, |\vec{p}\rangle e^{i\vec{r}\cdot(\vec{p}-\vec{p}')},$$

$$= |\mathcal{N}|^2 \int d^3\vec{p} \int d^3\vec{p}' F\left(-\left(\mathbf{p}'_\perp - \mathbf{p}_\perp\right)^2\right) e^{i\mathbf{b}_\perp\cdot(\mathbf{p}_\perp - \mathbf{p}'_\perp)}$$

$$\hookrightarrow \boxed{\rho(\vec{r}) = \int \frac{d^3\vec{\Delta}}{(2\pi)^3} F(-\vec{\Delta}^2) e^{-i\vec{r}_\perp\cdot\Delta_\perp}}$$

back

density interpretation of $q(x,\mathbf{b}_\perp)$

- express quark-bilinear in twist-2 GPD in terms of light-cone 'good' component $\psi_{(+)} \equiv \frac{1}{2}\gamma^-\gamma^+\psi$

$$\bar{\psi}'\gamma^+\psi = \bar{\psi}'_{(+)}\gamma^+\psi_{(+)} = \sqrt{2}\psi'^{\dagger}_{(+)}\psi_{(+)}.$$

- expand $\psi_{(+)}$ in terms of canonical raising and lowering operators

$$\psi_{(+)}(x^-,\mathbf{x}_\perp) = \int_0^\infty \frac{dk^+}{\sqrt{4\pi k^+}} \int \frac{d^2\mathbf{k}_\perp}{2\pi} \sum_s$$

$$\times \left[u_{(+)}(k,s)b_s(k^+,\mathbf{k}_\perp)e^{-ikx} + v_{(+)}(k,s)d_s^\dagger(k^+,\mathbf{k}_\perp)e^{ikx}\right]$$

with usual (canonical) equal light-cone time x^+ anti-commutation relations, e.g.

$$\left\{b_r(k^+, \mathbf{k}_\perp), b_s^\dagger(q^+, \mathbf{q}_\perp)\right\} = \delta(k^+ - q^+)\delta(\mathbf{k}_\perp - \mathbf{q}_\perp)\delta_{rs}$$

and the normalization of the spinors is such that

$$\bar{u}_{(+)}(p, r)\gamma^+ u_{(+)}(p, s) = 2p^+\delta_{rs}.$$

Note: $\bar{u}_{(+)}(p', r)\gamma^+ u_{(+)}(p, s) = 2p^+\delta_{rs}$ for $p^+ = p'^+$, one finds for $x > 0$

$$q(x, \mathbf{b}_\perp) = \mathcal{N}' \sum_s \int \frac{d^2\mathbf{k}_\perp}{2\pi} \int \frac{d^2\mathbf{k}'_\perp}{2\pi} \left\langle p^+, \mathbf{0}_\perp \right| b_s^\dagger(xp^+, \mathbf{k}'_\perp) b_s(xp^+, \mathbf{k}_\perp) \left| p^+, \mathbf{0}_\perp \right\rangle \times e^{i\mathbf{b}_\perp \cdot (\mathbf{k}_\perp - \mathbf{k}'_\perp)}.$$

- Switch to mixed representation:
  momentum in longitudinal direction
  position in transverse direction

$$\tilde{b}_s(k^+,\mathbf{x}_\perp) \equiv \int \frac{d^2\mathbf{k}_\perp}{2\pi} b_s(k^+,\mathbf{k}_\perp) e^{i\mathbf{k}_\perp\cdot\mathbf{x}_\perp}$$

$\hookrightarrow$

$$\begin{aligned} q(x,\mathbf{b}_\perp) &= \sum_s \left\langle p^+,\mathbf{0}_\perp \right| \tilde{b}_s^\dagger(xp^+,\mathbf{b}_\perp)\tilde{b}_s(xp^+,\mathbf{b}_\perp) \left| p^+, \right. \\ &= \sum_s \left| \tilde{b}_s(xp^+,\mathbf{b}_\perp) \left| p^+,\mathbf{0}_\perp \right\rangle \right|^2 \\ &\geq 0. \end{aligned}$$

back

Boosts in nonrelativistic QM

$$\vec{x}' = \vec{x} + \vec{v}t \qquad\qquad t' = t$$

purely kinematical (quantization surface $t = 0$ inv.)

$\hookrightarrow$ 1. boosting wavefunctions very simple

$$\Psi_{\vec{v}}(\vec{p}_1, \vec{p}_2) = \Psi_{\vec{0}}(\vec{p}_1 - m_1\vec{v}, \vec{p}_2 - m_2\vec{v}).$$

2. dynamics of center of mass

$$\vec{R} \equiv \sum_i x_i \vec{r}_i \qquad \text{with} \qquad x_i \equiv \frac{m_i}{M}$$

decouples from the internal dynamics

Relativistic Boosts

$$t' = \gamma\left(t + \frac{v}{c^2} z\right), \qquad z' = \gamma\,(z + vt) \qquad \mathbf{x}'_\perp = \mathbf{x}_\perp$$

generators satisfy Poincaré algebra:

$$\begin{aligned}
[P^\mu, P^\nu] &= 0 \\
[M^{\mu\nu}, P^\rho] &= i\,(g^{\nu\rho} P^\mu - g^{\mu\rho} P^\nu) \\
\left[M^{\mu\nu}, M^{\rho\lambda}\right] &= i\left(g^{\mu\lambda} M^{\nu\rho} + g^{\nu\rho} M^{\mu\lambda} - g^{\mu\rho} M^{\nu\lambda} - g^{\nu\lambda} M^{\mu\rho}\right)
\end{aligned}$$

rotations: $M_{ij} = \varepsilon_{ijk} J_k$, boosts: $M_{i0} = K_i$.

Galilean subgroup of $\perp$ boosts

introduce generator of $\perp$ 'boosts':

$$B_x \equiv M^{+x} = \frac{K_x + J_y}{\sqrt{2}} \qquad B_y \equiv M^{+y} = \frac{K_y - J_x}{\sqrt{2}}$$

Poincaré algebra $\Longrightarrow$ commutation relations:

$$\begin{aligned} [J_3, B_k] &= i\varepsilon_{kl} B_l & \qquad [P_k, B_l] &= -i\delta_{kl} P^+ \\ \left[P^-, B_k\right] &= -iP_k & \qquad [P^+, B_k] &= 0 \end{aligned}$$

with $k, l \in \{x, y\}$, $\varepsilon_{xy} = -\varepsilon_{yx} = 1$, and $\varepsilon_{xx} = \varepsilon_{yy} = 0$.

Together with $[J_z, P_k] = i\varepsilon_{kl}P_l$, as well as

$$\begin{aligned} \left[P^-, P_k\right] &= \left[P^-, P^+\right] = \left[P^-, J_z\right] = 0 \\ \left[P^+, P_k\right] &= \left[P^+, B_k\right] = \left[P^+, J_z\right] = 0. \end{aligned}$$

Same as commutation relations among generators of nonrel. boosts, translations, and rotations in x-y plane, provided one identifies

$$\begin{aligned} P^- &\longrightarrow \text{Hamiltonian} \\ \mathbf{P}_\perp &\longrightarrow \text{momentum in the plane} \\ P^+ &\longrightarrow \text{mass} \\ L_z &\longrightarrow \text{rotations around } z\text{-axis} \\ \mathbf{B}_\perp &\longrightarrow \text{generator of boosts in the plane,} \end{aligned}$$

back to discussion

Consequences

- many results from NRQM carry over to $\perp$ boosts in IMF, e.g.
- $\perp$ boosts kinematical

$$\begin{aligned} \Psi_{\Delta_\perp}(x, \mathbf{k}_\perp) &= \Psi_{\mathbf{0}_\perp}(x, \mathbf{k}_\perp - x\Delta_\perp) \\ \Psi_{\Delta_\perp}(x, \mathbf{k}_\perp, y, \mathbf{l}_\perp) &= \Psi_{\mathbf{0}_\perp}(x, \mathbf{k}_\perp - x\Delta_\perp, y, \mathbf{l}_\perp - y\Delta_\perp) \end{aligned}$$

- Transverse center of momentum $\mathbf{R}_\perp \equiv \sum_i x_i \mathbf{r}_{\perp,i}$ plays role similar to NR center of mass, e.g. $|\psi_{loc}\rangle \equiv \int d^2\mathbf{p}_\perp |p^+, \mathbf{p}_\perp\rangle$ corresponds to state with $\mathbf{R}_\perp = \mathbf{0}_\perp$.

back

$\perp$ Center of Momentum

- field theoretic definition

$$p^+\mathbf{R}_\perp \equiv \int dx^- \int d^2\mathbf{x}_\perp T^{++}(x)\mathbf{x}_\perp = M^{+\perp}$$

- $M^{+\perp} = \mathbf{B}^\perp$ generator of transverse boosts
- parton representation:

$$\mathbf{R}_\perp = \sum_i x_i \mathbf{r}_{\perp,i}$$

($x_i =$ momentum fraction carried by i^{th} parton)

back

Poincaré algebra:

$$
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back

Proof that $\mathbf{B}_\perp|p^+, \mathbf{R}_\perp = \mathbf{0}_\perp\rangle = 0$

- Use

$$e^{-i\mathbf{v}_\perp\cdot\mathbf{B}_\perp}|p^+,\mathbf{p}_\perp,\lambda\rangle = |p^+,\mathbf{p}_\perp + p^+\mathbf{v}_\perp,\lambda\rangle$$

$\hookrightarrow$

$$e^{-i\mathbf{v}_\perp\cdot\mathbf{B}_\perp}\int d^2\mathbf{p}_\perp|p^+,\mathbf{p}_\perp,\lambda\rangle = \int d^2\mathbf{p}_\perp|p^+,\mathbf{p}_\perp,\lambda\rangle$$

$\hookrightarrow$

$$\mathbf{B}_\perp\int d^2\mathbf{p}_\perp|p^+,\mathbf{p}_\perp,\lambda\rangle = 0$$

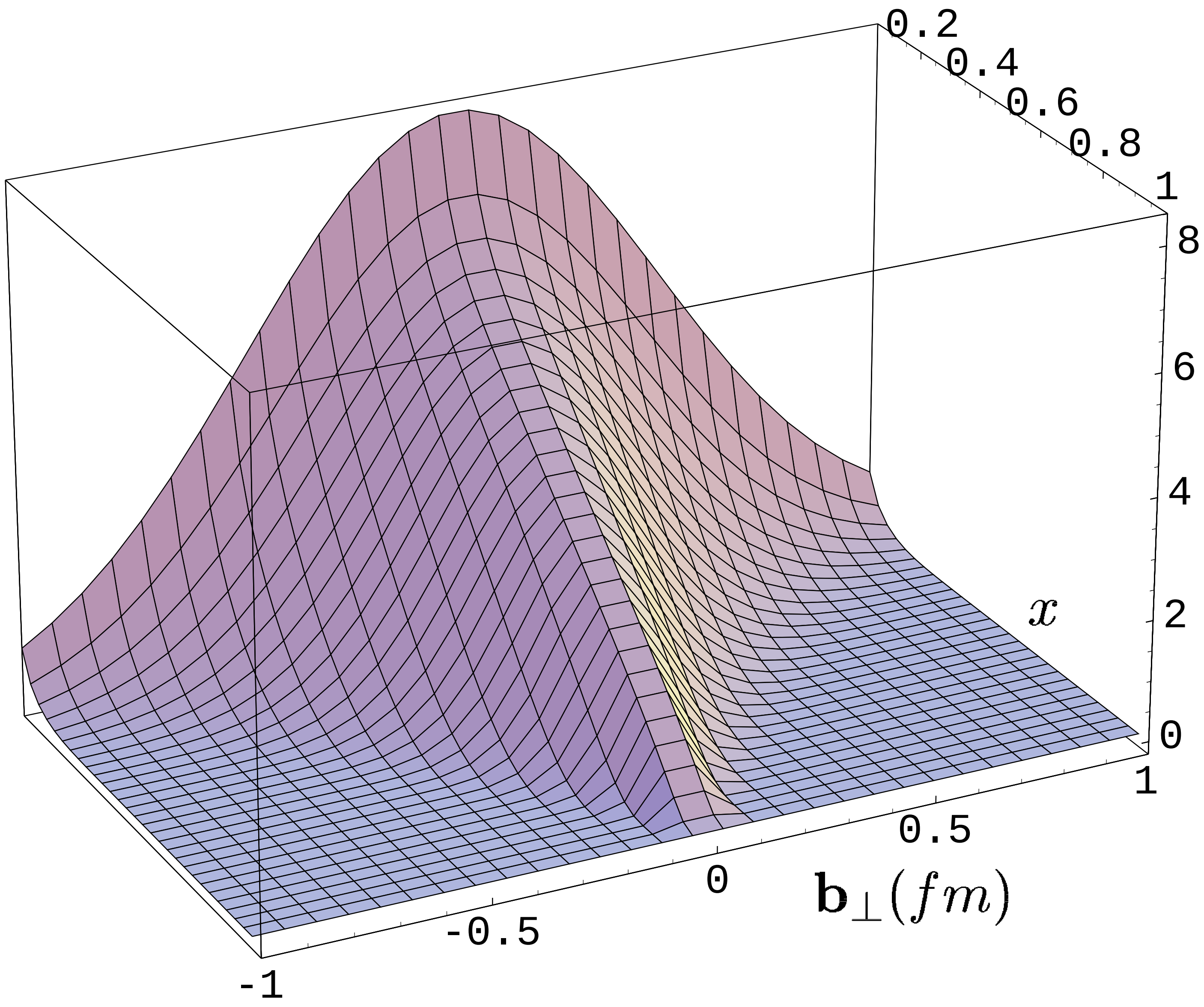
back

Example

- Ansatz: $H_q(x,0,-\Delta_\perp^2) = q(x)e^{-a\Delta_\perp^2(1-x)\ln\frac{1}{x}}$.

$$\hookrightarrow \quad q(x,\mathbf{b}_\perp^2) = q(x)\frac{1}{4\pi a(1-x)\ln\frac{1}{x}}e^{-\frac{\mathbf{b}_\perp^2}{4a(1-x)\ln\frac{1}{x}}}$$

$q(x, \mathbf{b}_\perp)$



back