LIGHT-FRONT QUARK MODEL ANALYSIS OF RARE $B \to K\ell^+\ell^-$ DECAYS

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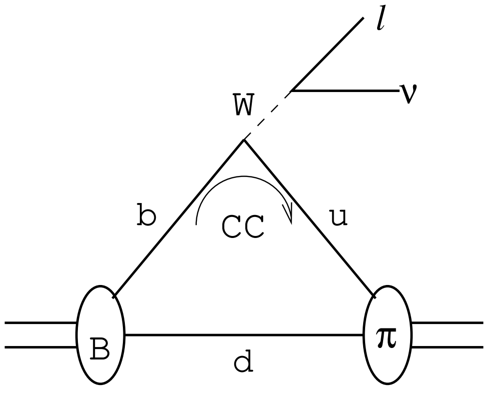
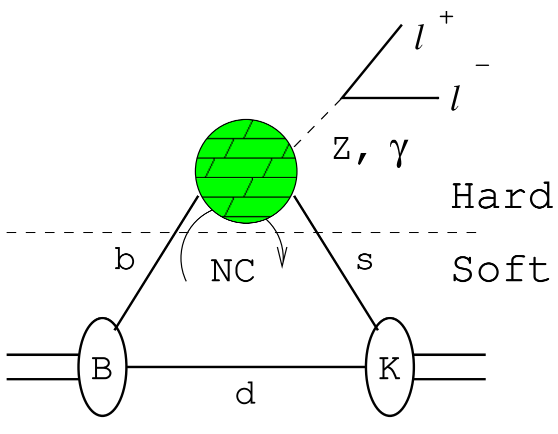
• B-factories(BaBar at SLAC, Belle at KEK, LHCB at CERN, B-TeV at Fermilab and CLEO at Cornell...) make precision test of standard model(SM) and beyond SM ever more promising!

• Accurate theoretical analysis of B-decays are thus strongly demanded!

• Especially, exclusive rare B-decays induced by the flavor-changing neutral current(FCNC) transition are well suited to test the SM and search for physics beyond the SM, SINCE



Flavor changing
charged current(CC)



Flavor changing
neutral current(FCNC)

⋆ One of the obstacles in theoretical understanding of exclusive decays is due to the nonperturbative hadronic form factor calculations.

→ Quark Model, Heavy Quark Theory, QCD Sum-Rules, Chiral Perturbation Theory, and Dyson-Shwinger Equation ...

• Outline

1. Overview of Effective Hamiltonian for $b \to s\ell^+\ell^-$ in the Standard Model(SM)

   (a) Short distance contribution
   (b) Long distance contribution

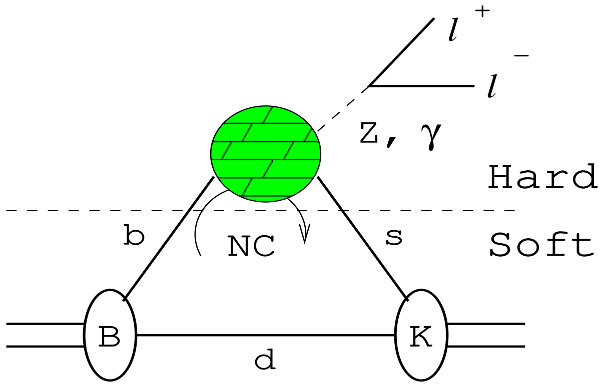
2. Light Front(LF) Formulation

3. Calculation of Hadronic Form Factors for $B \to K\ell^+\ell^-$ in LF Quark Model(LFQM)

4. Numerical Results

5. Conclusion

• Effective Hamiltonian for $B \to K\ell^+\ell^-$

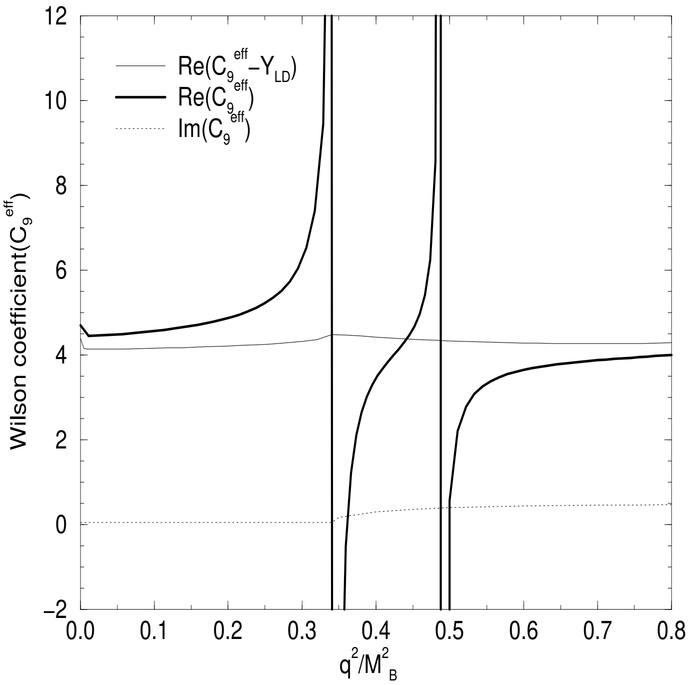


$$\mathcal{H}_{\rm eff}^{\ell^+\ell^-} = \frac{4G_F}{\sqrt{2}}\frac{e^2}{16\pi^2}V_{ts}^*V_{tb}\Big[-\frac{2iC_7(m_b)m_b}{q^2}\bar{s}\sigma_{\mu\nu}q^\nu P_R b\bar{\ell}\gamma^\mu\ell + C_9^{\rm eff}(m_b)\bar{s}\gamma_\mu P_L b\bar{\ell}\gamma^\mu\ell + C_{10}(m_b)\bar{s}\gamma_\mu P_L b\bar{\ell}\gamma^\mu\gamma_5\ell\Big],$$

where the effective Wilson coefficient $C_9^{\rm eff}(\hat{s}{=}q^2/m_b^2)$ is given by

$$C_9^{\rm eff}(\hat{s}) \equiv \underbrace{C_9^{\rm nr}(\hat{s})}_{\text{Non-resonant contributions}} + \underbrace{Y_{\rm LD}(\hat{s})}_{J/\psi,\psi',\dots\text{ resonances}},$$

[T. M. Aliev et al., Phys. Lett. B441, 410(98); Z. Ligeti and M. B. Wise, Phys. Rev. D53, 4937(96).]



• Long-distance contribution to $B \to K\ell^+\ell^-$

(i) Matrix elements of the hadronic currents for $B \to K$ transition:

$$J^\mu \equiv \langle K|\bar{s}\gamma^\mu P_L b|B\rangle = \frac{1}{2}[F_+(q^2)P^\mu + F_-(q^2)q^\mu],$$

and

$$J_T^\mu \equiv \langle K|\bar{s}i\sigma^{\mu\nu}q_\nu P_R b|B\rangle = \frac{1}{2(M_B+M_K)}[q^2 P^\mu - (M_B^2 - M_K^2)q^\mu]F_T(q^2),$$

where $P = P_B + P_K$, $q = P_B - P_K$ and $4m_l^2 \le q^2 \le (M_B - M_K)^2$. The form factors $F_\pm$ satisfy

$$F_+(0) = F_0(0),\ F_0(q^2) = F_+(q^2) + \frac{q^2}{M_B^2 - M_K^2}F_-(q^2).$$

(ii) Transition amplitude:

$$\begin{aligned}\mathcal{M} &= \langle K\ell^+\ell^-|\mathcal{H}_{\rm eff}|B\rangle \\ &= \frac{4G_F}{\sqrt{2}}\frac{\alpha}{4\pi}V_{ts}^*V_{tb}\Big\{\Big[C_9^{\rm eff}J_\mu - \frac{2m_b}{q^2}C_7 J_\mu^T\Big]\bar{\ell}\gamma^\mu\ell + C_{10}J_\mu\bar{\ell}\gamma^\mu\gamma_5\ell\Big\}.\end{aligned}$$

(iii) Differential decay rate:

$$\frac{d\Gamma}{d\hat{s}} = \frac{M_B^5 G_F^2}{3\cdot 2^9\pi^5}\alpha^2|V_{ts}^*V_{tb}|^2\hat{\phi}^{1/2}\Big(1 - 4\frac{\hat{m}_\ell}{\hat{s}}\Big)^{1/2}\Big[\hat{\phi}\Big(1 + 2\frac{\hat{m}_\ell}{\hat{s}}\Big)F_{T+} + 6\frac{\hat{m}_\ell}{\hat{s}}F_{0+}\Big],$$

where

$$\begin{aligned}F_{T+} &= |C_9^{\rm eff}F_+ - \frac{2C_7}{1+\sqrt{\hat{r}}}F_T|^2 + |C_{10}|^2|F_+|^2, \\ F_{0+} &= |C_{10}|^2[(1-\hat{r})^2|F_0|^2 - \hat{\phi}|F_+|^2], \\ \hat{\phi} &= (\hat{s} - 1 - \hat{r})^2 - 4\hat{r},\end{aligned}$$

with $\hat{s} = q^2/M_B^2$, $\hat{m}_\ell = m_\ell^2/M_B^2$, and $\hat{r} = M_K^2/M_B^2$.

(iv) Longitudinal lepton polarization asymmetry(LPA)

$$\begin{aligned}P_L(\hat{s}) &= \frac{d\Gamma_{h=-1}/d\hat{s} - d\Gamma_{h=1}/d\hat{s}}{d\Gamma_{h=-1}/d\hat{s} + d\Gamma_{h=1}/d\hat{s}} \\ &= \frac{2\Big(1 - 4\frac{\hat{m}_\ell}{\hat{s}}\Big)^{1/2}\hat{\phi}C_{10}F_+\Big[F_+{\rm Re}C_9^{\rm eff} - \frac{2C_7}{1+\sqrt{\hat{r}}}F_T\Big]}{\Big[\hat{\phi}\Big(1 + 2\frac{\hat{m}_\ell}{\hat{s}}\Big)F_{T+} + 6\frac{\hat{m}_\ell}{\hat{s}}F_{0+}\Big]}.\end{aligned}$$

• What is special of our method?

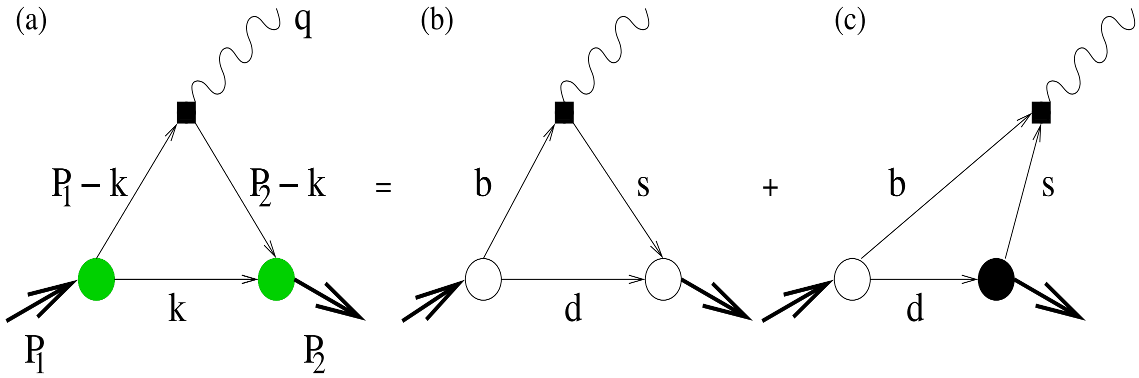


Fig.1. The covariant diagram (a) corresponds to the sum of the LF valence (b) and the nonvalence (c) diagrams.

(A) In $q^+ = 0 (q^2 = q^+q^- - \vec{q}^2 < 0)$ frame:

F_+ and F_T need only valence diagram (b), i.e. No zero-mode!
F_-(and F_0) need both valence (b) and zero-mode contributions!

(B) In $q^+ \neq 0 (q^2 > 0)$ frame:

F_+, $F_-(F_0)$ and F_T needs both diagrams (b) and (c)!

Previous Works done by others

• In $q^+ = 0$ frame: Obtained only F_+ and F_T(but not F_-) and used parametric formula

$$F(q^2) = \frac{F(0)}{1 - \sigma_1 q^2 + \sigma_2 q^4},$$

to extrapolate to the timelike region.

• In $q^+ \neq 0$ frame: Obtained F_+, F_- and F_T, BUT using only val. diagram(b).

Our Method

(I) F_+ and F_T in $q^+ = 0$ frame and then analytically continue to the timelike region by changing $\vec{q}_\perp$ to i$\vec{q}_\perp$.

(II) $F_-(F_0)$ in $q^+ \neq 0$ frame using the effective treatment of the nonvalence diagram (c) based on the Bethe-Salpeter formalism.

• LFQM calculation of F_+ and F_T in $q^+ = 0$ frame

Matrix elements of the currents J^μ and J^μ_T for $P_B(q_1\bar{q}) \to P_K(q_2\bar{q})$ transitions:

$$\langle P_2|\bar{q}_2\Gamma^\mu q_1|P_1\rangle = \sum_{\lambda' s}\int d^3\vec{p}_{\bar{q}}\left[\phi_2(x,\vec{k}'_\perp)\mathcal{R}^{00\dagger}_{\lambda_2\bar{\lambda}}\right]\frac{\bar{u}_{\lambda_2}(p_2)}{\sqrt{p_2^+}}\Gamma^\mu\frac{u_{\lambda_1}(p_1)}{\sqrt{p_1^+}}\left[\mathcal{R}^{00}_{\lambda_1\bar{\lambda}}\phi_1(x,\vec{k}_\perp)\right],$$

where $\Gamma^\mu = \gamma^\mu P_L$ for J^μ and $i\sigma^{\mu\nu}q_\nu P_R$ for J^μ_T, respectively.

The spin-orbit wave function $\mathcal{R}^{JJ_z}_{\lambda_q,\lambda_{\bar{q}}}(x,\vec{k}_\perp)$ and the radial wave function $\phi(x,\vec{k}_\perp)$ are given by

$$\mathcal{R}^{J=0,J_z=0}_{\lambda_q,\lambda_{\bar{q}}}(x,\vec{k}_\perp) = \frac{\bar{u}(p_q,\lambda_q)\gamma^5 v(p_{\bar{q}},\lambda_{\bar{q}})}{\sqrt{2}\sqrt{M_0^2-(m_q^2-m_{\bar{q}}^2)^2}},\ M_0^2 = \frac{m_q^2+\vec{k}_\perp^2}{1-x}+\frac{m_{\bar{q}}^2+\vec{k}_\perp^2}{x},$$

$$\phi(x,\vec{k}_\perp) = \left(\frac{1}{\pi^{3/2}\beta^3}\right)^{1/2}\exp(-\vec{k}^2/2\beta^2).$$

Explicitly, the weak form factors $F_+(\vec{q}^2_\perp)$ and $F_T(\vec{q}^2_\perp)$ are obtained as

$$F_+(\vec{q}^2_\perp) = \int_0^1\frac{dx}{x(1-x)}\int d^2\vec{k}_\perp\sqrt{\frac{\partial k'_z}{\partial x}}\sqrt{\frac{\partial k_z}{\partial x}}\phi_2(x,\vec{k}'_\perp)\phi_1(x,\vec{k}_\perp) \times\frac{[xm_1+(1-x)m_{\bar{q}}][xm_2+(1-x)m_{\bar{q}}]+\vec{k}_\perp\cdot\vec{k}'_\perp}{\sqrt{M_0^2-(m_1-m_{\bar{q}})^2}\sqrt{M_0'^2-(m_2-m_{\bar{q}})^2}},$$

and

$$F_T(\vec{q}^2_\perp) = -\int_0^1\frac{dx}{(1-x)}\int d^2\vec{k}_\perp\sqrt{\frac{\partial k'_z}{\partial x}}\sqrt{\frac{\partial k_z}{\partial x}}\phi_2(x,\vec{k}'_\perp)\phi_1(x,\vec{k}_\perp) \times\frac{(M_B+M_K)\left[(m_2-m_1)\frac{\vec{k}_\perp\cdot\vec{q}_\perp}{\vec{q}^2_\perp}+xm_1+(1-x)m_{\bar{q}}\right]}{\sqrt{M_0^2-(m_1-m_{\bar{q}})^2}\sqrt{M_0'^2-(m_2-m_{\bar{q}})^2}},$$

where $\vec{k}'_\perp = \vec{k}_\perp - x\vec{q}_\perp$.

To obtain timelike form factors for F_+ and F_T, we use the analytic continuation by replacing $q_\perp$ with $iq_\perp$ in the form factors.

• Covariant Bethe-Salpeter(BS) amplitude of the current $J^\mu(0)$

[see C.-R. Ji and H.-M. Choi, Phys. Lett. B513, 330(01); H.-M. Choi, C.-R. Ji and L. S. Kisslinger, Phys. Rev. D64, 093006(01)]

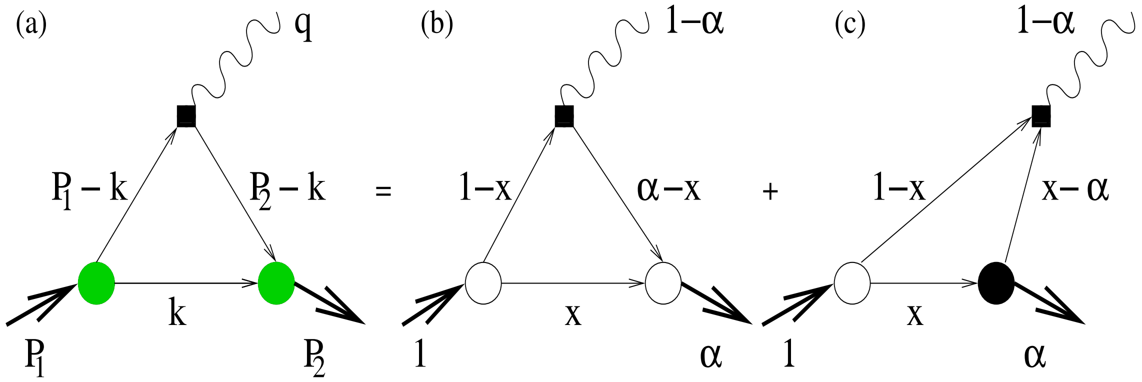


Fig.2. $x = k^+/P_1^+$ and $\alpha = P_2^+/P_1^+ = 1 - q^+/P_1^+$.

Matrix element of the hadronic current J^μ:

$$J^\mu = \langle K(P_2)|\gamma^\mu P_L|B(P_1)\rangle = \frac{1}{2}\Big[F_+(q^2)(P_1+P_2)^\mu + F_-(q^2)q^\mu\Big],$$

where $q^\mu = (P_1 - P_2)^\mu$ and $m_\ell^2 \le q^2 \le (M_1 - M_2)^2$.

$$\begin{aligned} J^\mu_{\rm cov} &= iN_c \int \frac{d^4k}{(2\pi)^4} \frac{H_1^{\rm cov} S^\mu H_2^{\rm cov}}{(p_1^2 - m_1^2 + i\epsilon)(p_2^2 - m_2^2 + i\epsilon)(p_{\bar q}^2 - m_{\bar q}^2 + i\epsilon)} \\ &= J^\mu_{\rm val} + J^\mu_{\rm nv}, \end{aligned}$$

where $p_1 = P_1 - k$, $p_2 = P_2 - k$, and $p_{\bar q} = k$ and

$$S^\mu = {\rm Tr}[\gamma_5(\not p_1 + m_1)\gamma^\mu P_L(\not p_2 + m_2)\gamma_5(-\not p_{\bar q} + m_{\bar q})]$$

- Pole structure(over k^-) in LF calculation ($d^4k = dk^+dk^-d^2k_\perp/2$):

$$(i)\ \ k_q^- = \frac{k_\perp^2 + m_{\bar q}^2 - i\epsilon}{k^+}$$

$$(ii)\ \ k_1^- = P_1^- - \frac{(k_\perp - P_{1\perp})^2 + m_1^2 - i\epsilon}{P_1^+ - k^+}$$

$$(iii)\ \ k_2^- = P_2^- - \frac{(k_\perp - P_{2\perp})^2 + m_2^2 - i\epsilon}{P_2^+ - k^+}$$

(I) Valence Contribution($0 \le k^+ \le P_2^+$) with $k^- = k_q^-$-Pole:

$$J^+_{\rm val} = \frac{N_c}{16\pi^3}\int_0^\alpha \frac{dx}{x(1-x)(1-x')}\int d^2\vec{k}_\perp \chi_{(2\to 2)}(x,\vec{k}_\perp) S^+_{\rm val}\chi'_{(2\to 2)}(x',\vec{k}'_\perp),$$

where $x' = x/\alpha$, $\vec{k}'_\perp = \vec{k}_\perp - x'\vec{q}_\perp$ and

$$\chi_{(2\to 2)}(x,\vec{k}_\perp) = \frac{h_1^{\rm LF}}{(M_1^2 - M_0^2)},\ M_0^2 = \frac{m_1^2 + \vec{k}^2_\perp}{1-x} + \frac{m_{\bar q}^2 + \vec{k}^2_\perp}{x},$$

$$\chi'_{(2\to 2)}(x',\vec{k}'_\perp) = \frac{h_2^{\rm LF}}{(M_2^2 - M_0'^2)},\ M_0'^2 = \frac{m_2^2 + \vec{k}'^2_\perp}{1-x'} + \frac{m_{\bar q}^2 + \vec{k}'^2_\perp}{x'},$$

$$S^+_{\rm val} = \left(\frac{2P_1^+}{x'}\right)\left\{\vec{k}_\perp\cdot\vec{k}'_\perp + [m_1 x + m_{\bar q}(1-x)][m_2 x' + m_{\bar q}(1-x')]\right\}$$

Now, as $q^+ \to 0 (P_1^+ = P_2^+)$, one gets $\alpha \to 1$, $x' \to x$, $J^+_{\rm val} = P_1^+ F_+$ and $J^+_{\rm nv} \to 0$, i.e.

$$F_+^{LFBS}(\vec{q}^2_\perp) = \frac{N_c}{8\pi^3}\int_0^1 \frac{dx}{x(1-x)}\int d^2\vec{k}_\perp \chi_{(2\to 2)}(x,\vec{k}_\perp)\chi'_{(2\to 2)}(x,\vec{k}'_\perp)$$
$$\times\frac{\vec{k}_\perp\cdot\vec{k}'_\perp + [m_1 x + m_{\bar q}(1-x)][m_2 x + m_{\bar q}(1-x)]}{x(1-x)}.$$

Comparing this with direct calculation from our LFQM, which is given by

$$F_+^{LFQM}(\vec{q}^2_\perp) = \int_0^1 \frac{dx}{x(1-x)}\int d^2\vec{k}_\perp \sqrt{\frac{\partial k'_z}{\partial x}}\sqrt{\frac{\partial k_z}{\partial x}}\phi_2(x,\vec{k}'_\perp)\phi_1(x,\vec{k}_\perp)$$
$$\times\frac{[xm_1 + (1-x)m_{\bar q}][xm_2 + (1-x)m_{\bar q}] + \vec{k}_\perp\cdot\vec{k}'_\perp}{\sqrt{M_0^2 - (m_1 - m_{\bar q})^2}\sqrt{M_0'^2 - (m_2 - m_{\bar q})^2}},$$

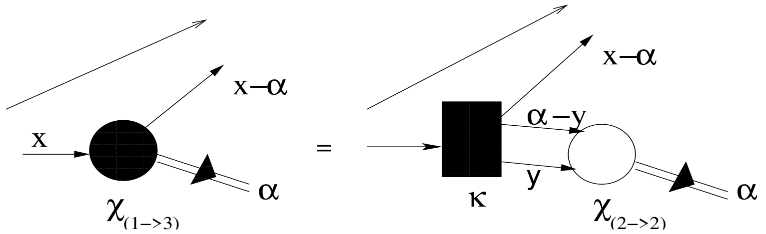
we identify what $\chi_{(2\to 2)}(x,\vec{k}_\perp)$ should be

$$\chi_{(2\to 2)}(x,\vec{k}_\perp) = \sqrt{\frac{8\pi^3}{N_c}}\sqrt{\frac{\partial k_z}{\partial x}}\frac{[x(1-x)]^{1/2}}{\sqrt{M_0^2 - (m_q - m_{\bar q})^2}}\phi(x,\vec{k}_\perp).$$

Tensor form factor, F_T, can be analyzed in a similar way to obtain F_+, **but** not for $F_-(F_0)$!!

How do we obtain F_- then ?

(II) Nonvalence Contribution ($P_2^+ \leq k^+ \leq P_1^+$) with $k^- = k_1^-$-Pole:



Using the LFBS equation

$$(M^2 - M_0'^2)\chi_{(1\to3)}(x_i', \vec{k}_{i\perp}') = \int [dy'][d^2\vec{l}_\perp']\mathcal{K}(x_i', \vec{k}_{i\perp}'; y_j', \vec{l}_{j\perp}')\chi_{(2\to2)}(y_j', \vec{l}_{j\perp}'),$$

we obtain

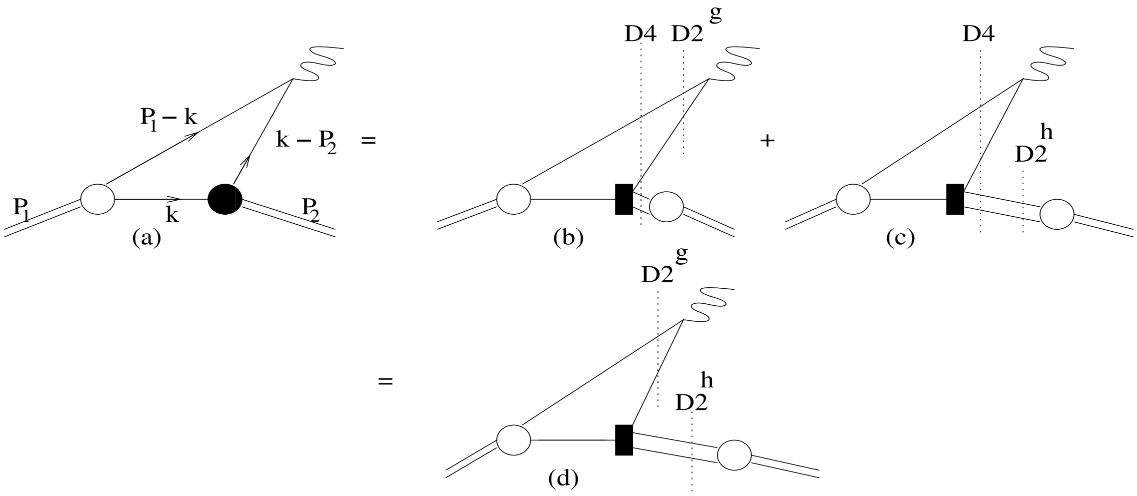
$$J_{\rm nv}^+ = \frac{-N_c}{16\pi^3}\int_\alpha^1 \frac{dx}{x(1-x)x''}\int d^2\vec{k}_\perp \chi_{(2\to2)}(x,\vec{k}_\perp) S_{\rm nv}^+ \chi^g(x'',\vec{k}_\perp'')$$
$$\times \underbrace{\int \frac{dy'}{y'(1-y')}\int d^2\vec{l}_\perp \mathcal{K}(x',\vec{k}_\perp'; y',\vec{l}_\perp')\chi_{(2\to2)}(y',\vec{l}_\perp'),}_{G(x,\vec{k}_\perp)}$$

where $y' = y/\alpha,\ \vec{l}_\perp' = \vec{l}_\perp - y'\vec{q}_\perp,\ x'' = (1-x)/(1-\alpha),\ \vec{k}_\perp'' = \vec{k}_\perp - x''\vec{q}_\perp$ and

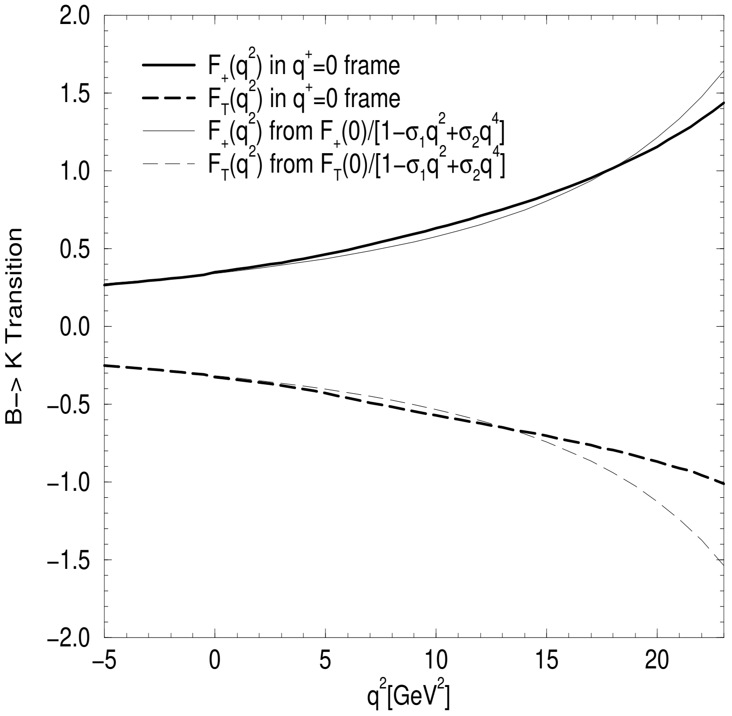
$$\chi^g(x,\vec{k}_\perp) = \frac{1}{\left[q^2 - \left(\frac{\vec{k}_\perp''^2 + m_1^2}{x''} + \frac{\vec{k}_\perp''^2 + m_2^2}{1-x''}\right)\right]},$$

$$S_{\rm nv}^+ = S_{\rm val}^+ + \frac{2P_1^+}{x'}x(1-x)(1-x')(M_1^2 - M_0^2),$$

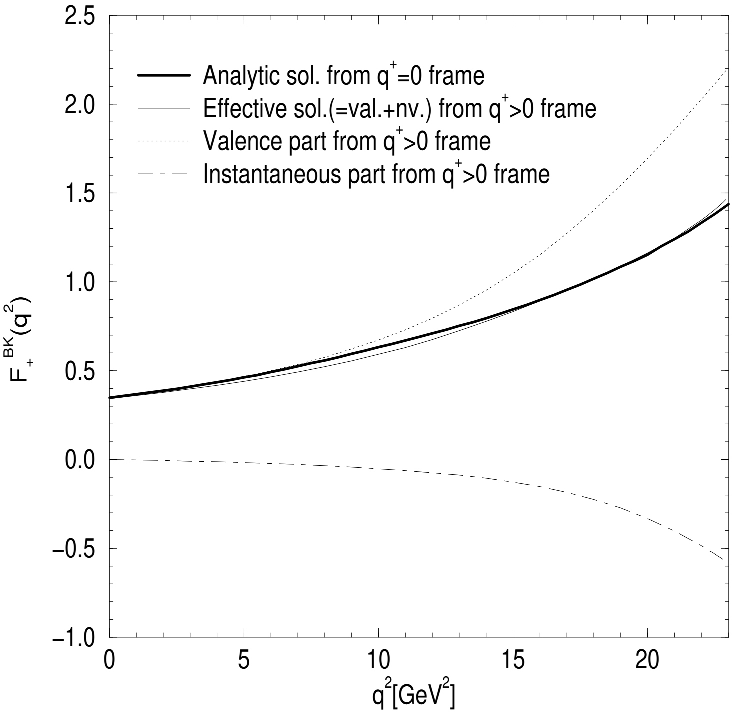
- Cancellation of the 4-Body Energy Denominator in LF Time-Ordered Diagrams:



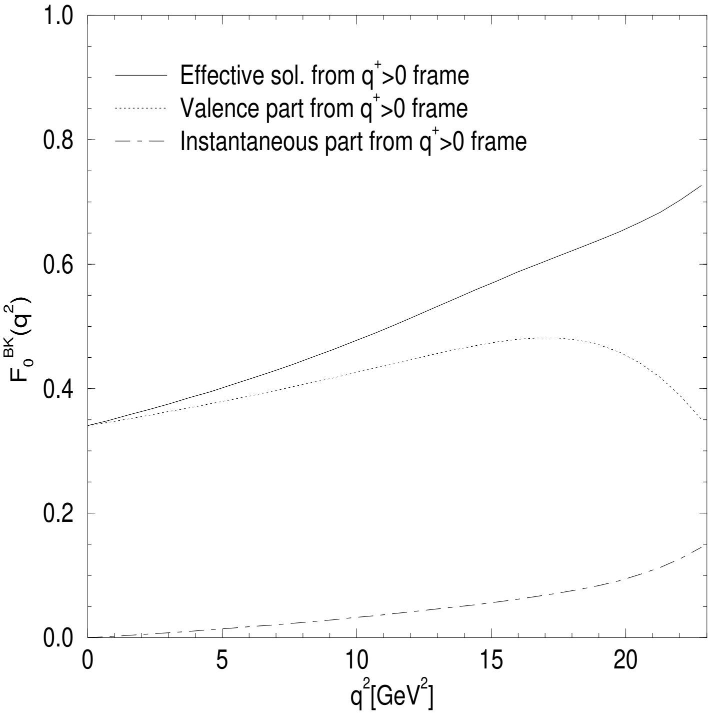
$$\frac{1}{D_4 D_2^g} + \frac{1}{D_4 D_2^h} = \frac{1}{D_2^g D_2^h} \to \chi^g(x,\mathbf{k}_\perp)\chi_{(2\to2)}(y,\mathbf{l}_\perp)$$



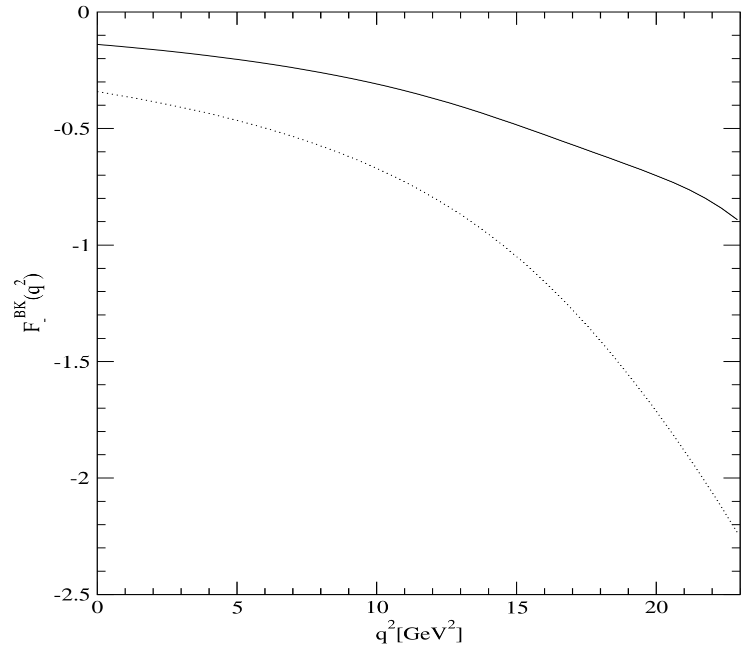
$F_+(q^2)$ in q^+=0 frame
$F_T(q^2)$ in q^+=0 frame
$F_+(q^2)$ from $F_+(0)/[1-\sigma_1 q^2+\sigma_2 q^4]$
$F_T(q^2)$ from $F_T(0)/[1-\sigma_1 q^2+\sigma_2 q^4]$
B−> K Transition
q^2[GeV^2]



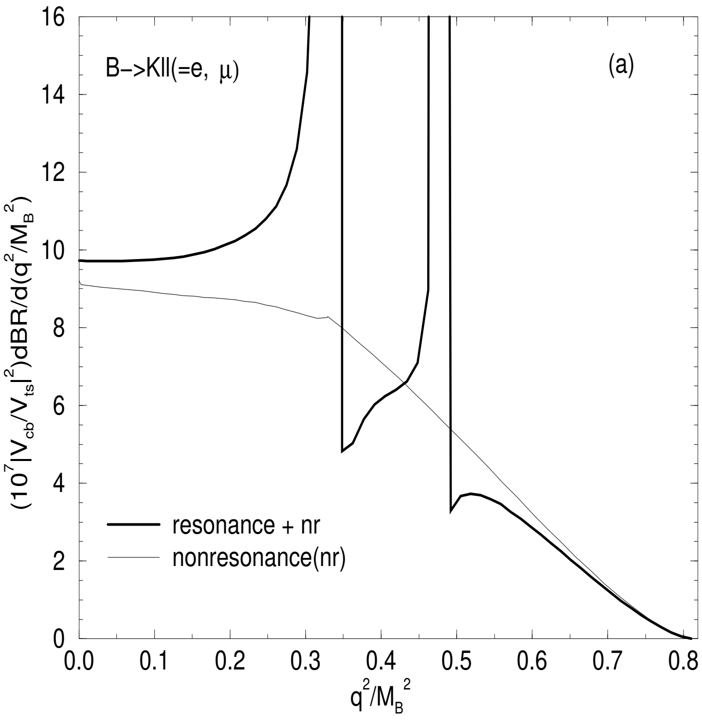
Analytic sol. from q^+=0 frame
Effective sol.(=val.+nv.) from q^+>0 frame
Valence part from q^+>0 frame
Instantaneous part from q^+>0 frame
$F_+^{BK}(q^2)$
q^2[GeV^2]



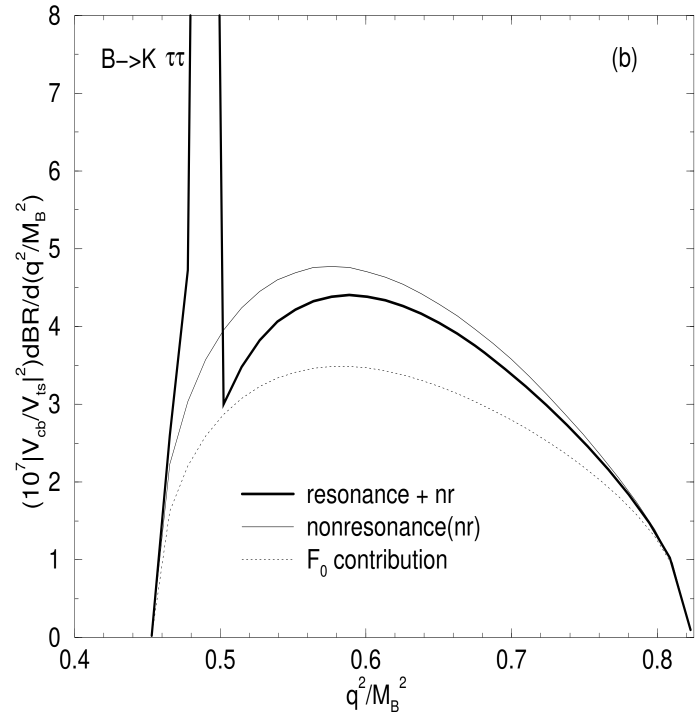
Effective sol. from q+>0 frame
Valence part from q+>0 frame
Instantaneous part from q+>0 frame
F0BK(q2)
q2[GeV2]



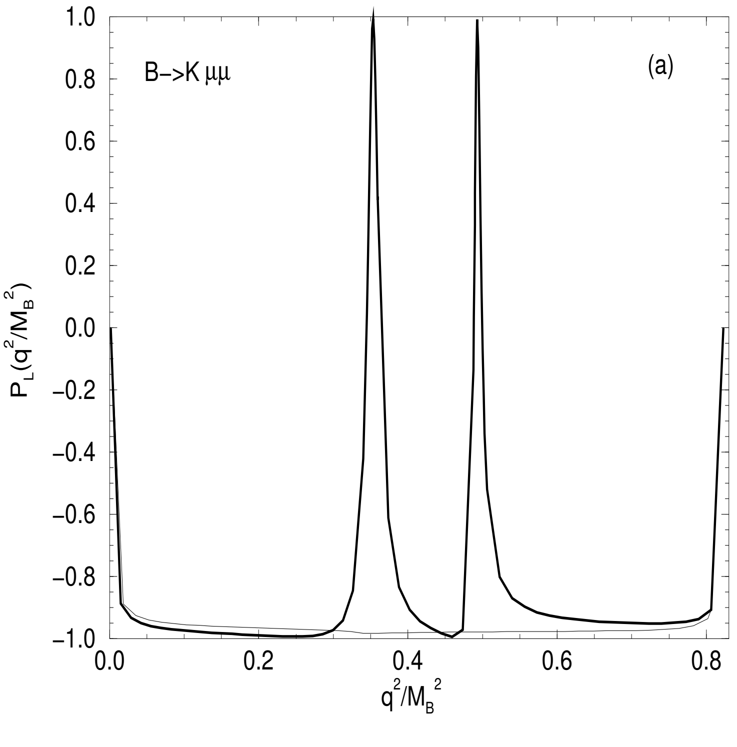
F-BK(q2)
q2[GeV2]

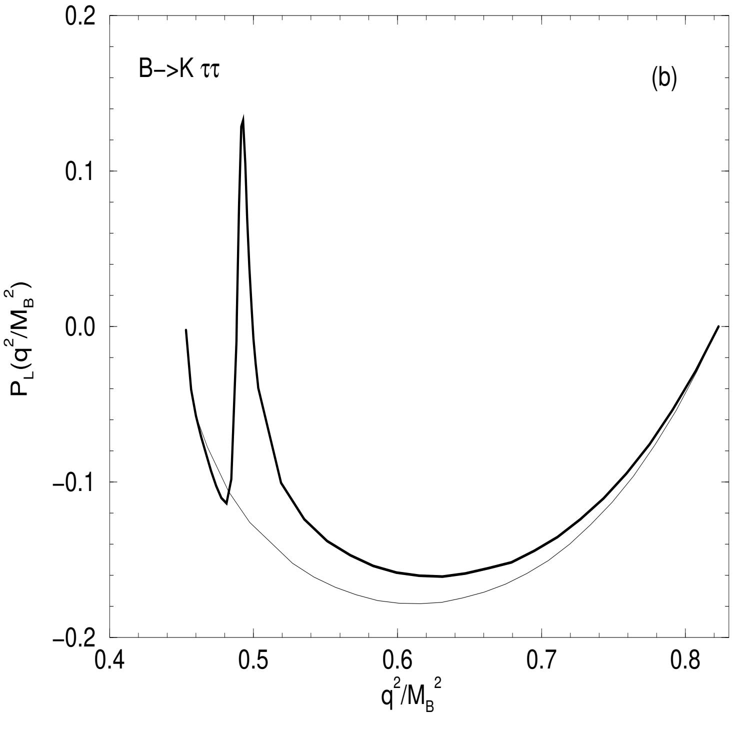


B->Kll(=e, μ)
(a)
resonance + nr
nonresonance(nr)
$(10^7|V_{cb}/V_{ts}|^2)dBR/d(q^2/M_B^2)$
q^2/M_B^2



B->K ττ
(b)
resonance + nr
nonresonance(nr)
F_0 contribution
$(10^7|V_{cb}/V_{ts}|^2)dBR/d(q^2/M_B^2)$
q^2/M_B^2


B−>K μμ
(a)
$P_L(q^2/M_B^2)$
q^2/M_B^2
1.0
0.8
0.6
0.4
0.2
0.0
−0.2
−0.4
−0.6
−0.8
−1.0
0.0
0.2
0.4
0.6
0.8


B−>K ττ
(b)
$P_L(q^2/M_B^2)$
q^2/M_B^2
0.2
0.1
0.0
−0.1
−0.2
0.4
0.5
0.6
0.7
0.8

• Conclusion

• We investigated the rare $B \to K\ell^-\ell^+$ decays within the SM, using our LFQM:

We

1. obtain F_+ and F_T in $q^+ = 0$ frame($q^2 < 0$) and then analytically continue them to the timelike region($q^2 > 0$)

2*. obtain $F_-(F_0)$ in $q^+ > 0$ frame($q^2 > 0$) using the effective treatment of the nonvalence diagram based on the BS formalism.

3. check the covariance(i.e. frame independence) of our model by comparing $F_+(q^2)$ obtained in $q^+ = 0$ frame and that in $q^+ > 0$ frame.

• Our numerical results for the form factors, decay rates, LPA are comparable with the available data as well as other theoretical model predictions:

1. Our numerical result for $\mathrm{BR}(B \to K\ell^+\ell^-)(\ell = e, \mu) = 0.5 \times 10^{-6}$ is in agreement with the data, $0.37 \times 10^{-6} \leq \mathrm{BR}^{\exp} \leq 1.2 \times 10^{-6}$.

2. more experimental data such as $\mathrm{BR}(B \to K\tau^+\tau^-)$ and LPA are still needed to make definitive test on existing model.

• Future work: $B \to K^*$?