Non perturbative renormalization in covariant light front dynamics

Thibaut Cousin (+)

Vladimir Karmanov (*)

(+)*Laboratoire de Physique Corpusculaire, Clermont-Ferrand, France*

(*)*Lebedev Physical Institute, Moscow, Russia*

- State vector in a scalar model
- Covariant light-front dynamics
- Physical problem
- 2-particle problem
- 3-particle problem
- Generalization
- Including spin. . .

State vector in a scalar model

Two scalar fields interacting as $g\psi^2\varphi$:

$\longrightarrow$ "Nucleon" field ψ of mass m

$\longrightarrow$ "Pion" field φ of mass μ

A nucleon state vector obeys:

$$\hat{P}^2 |\, p\rangle = M^2 |\, p\rangle$$

in which the nucleon is fully dressed with pions.

Can $|\, p\rangle$ be computed ?

Covariant light-front dynamics

J. Carbonell, B. Desplanques, V.A. Karmanov, J.-F. Mathiot,

Phys. Rep, **300** (1998) 215

- Why covariant?

$$t^+ = t + z \quad \Rightarrow \quad \sigma = \omega_\mu . x^\mu, \ \omega^2 = 0$$

Dependance on orientation of LF plane parametrized by 4-vector ω^μ

- Light-front vacuum:

$$P_0 > 0 \ \Leftrightarrow \ P_+, P_- > 0$$

$\Rightarrow$ light-front vacuum is trivial

$\Rightarrow$ diagrams are not spoiled by vacuum bubbles

$\Rightarrow$ Fock space analysis

- Fock decomposition:

$$| p\rangle = \phi_1 | N\rangle + \phi_2 | N\pi\rangle + \phi_3 | N\pi\pi\rangle + \ldots$$

The ϕ_i are the wave functions.

- Particles:

$\Rightarrow$ σ-ordered diagrams

$\Rightarrow$ all particles are always on-mass shell

$\Rightarrow$ the triviality of the vacuum removes many diagrams

Physical problem

$$\hat{P}^2 | p\rangle = M^2 | p\rangle, \qquad \hat{P}_\mu = \hat{P}^0_\mu + \hat{P}^{\text{int}}_\mu$$

$$\hat{P}^0_\mu = \sum_i \int d^3\vec{k}\ d_i^\dagger(\vec{k}) d_i(\vec{k})$$

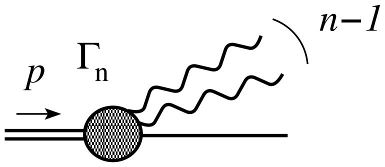
$$\hat{P}^{\text{int}}_\mu = \omega_\mu \int \frac{d\tau}{2\pi}\ \mathcal{H}_{\text{int}}(\tau\omega)$$

With $\left[\omega.\hat{P}_0, \mathcal{H}_{\text{int}}(\tau\omega)\right] = 0$ we get:

$$\left(\hat{P}_0^2 + 2(\omega.p) \int \frac{d\tau}{2\pi}\ \mathcal{H}_{\text{int}}(\tau\omega)\right) | p\rangle = M^2 | p\rangle$$

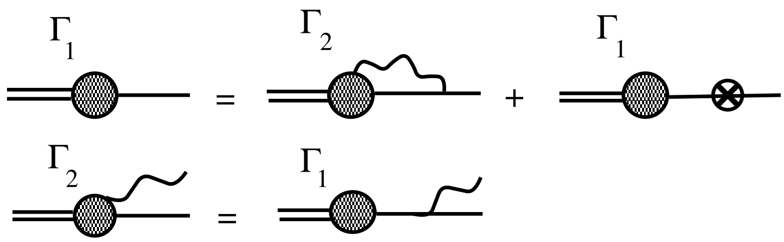
$$\boxed{\frac{1}{2\pi} \int \frac{d\tau}{\tau}\ \mathcal{H}_{\text{int}}(\tau\omega) | \mathcal{G}(p)\rangle = - | \mathcal{G}(p)\rangle}$$

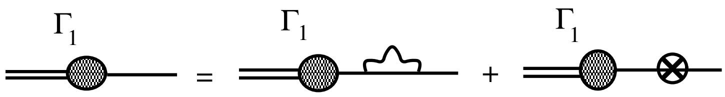
Fock components of $| \mathcal{G}(p)\rangle$:

$\Gamma_n = (s - m^2)\phi_n =$ 

2-particle problem

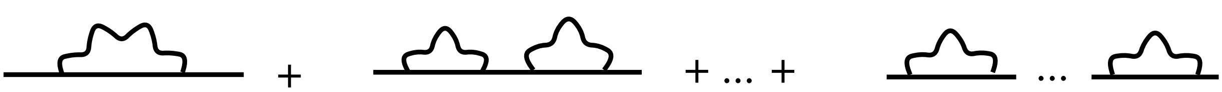
$$|\,p\rangle = \phi_1|\,N\rangle + \phi_2|\,N\pi\rangle$$



$\Rightarrow$ 

Trivialy yields $\delta m^2 = -g^2\Sigma(p^2 = m^2)$, as identical to the perturbative case.

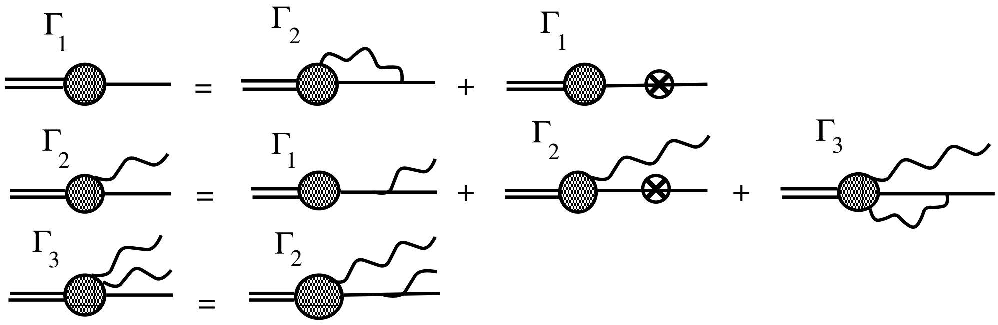
Iterating the relation:



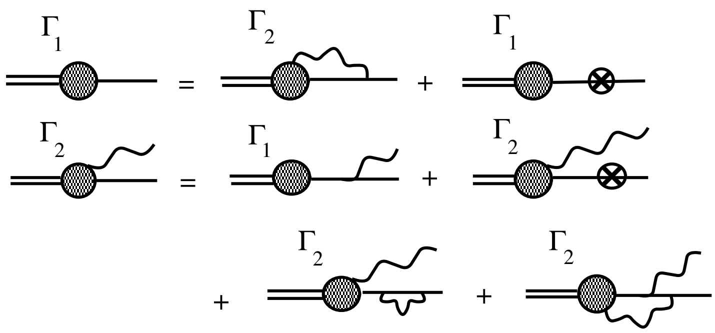
$\Rightarrow$ only one irreducible diagram

3-particle problem

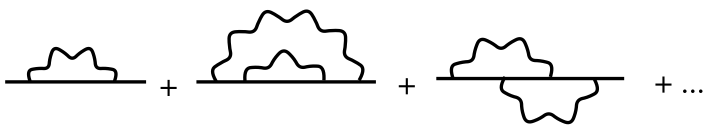
$$|\, p\rangle = \phi_1 |\, N\rangle + \phi_2 |\, N\pi\rangle + \phi_3 |\, N\pi\pi\rangle$$



$\Rightarrow$



Iterating the relation:



$\Rightarrow$ infinite number of irreducible diagrams

Equation giving Γ_1:

$$\frac{\delta m^2 + g^2\Sigma(s_1)}{(s-m^2)x_1}\Gamma_1(\vec{q},\vec{n})$$

$$-\frac{g^2}{\delta m^2}\int\frac{d^3\vec{q}'}{(2\pi)^3}\,\Sigma_i(\vec{q}',m^2)\Gamma_1(\vec{q}',\vec{n})$$

$$+g^2\int\frac{d^3\vec{q}'}{(2\pi)^3}\,\Sigma_i(\vec{q}',m^2)\Pi(\vec{q}',\vec{q},\vec{n},m^2)\Gamma_1(\vec{q}',\vec{n})$$

$$=\Gamma_1(\vec{q},\vec{n}) \qquad \equiv \lambda(\delta m^2)\Gamma_1(\vec{q},\vec{n})$$

- δm^2 is determined by condition $\lambda(\delta m^2) = 1$
- δm^2 differs from its perturbative value δm_0^2

Numerical solution

D. Bernard, T. Cousin, V.A. Karmanov, J.-F. Mathiot, Phys. Rev. D65, p. 025016 (2002)

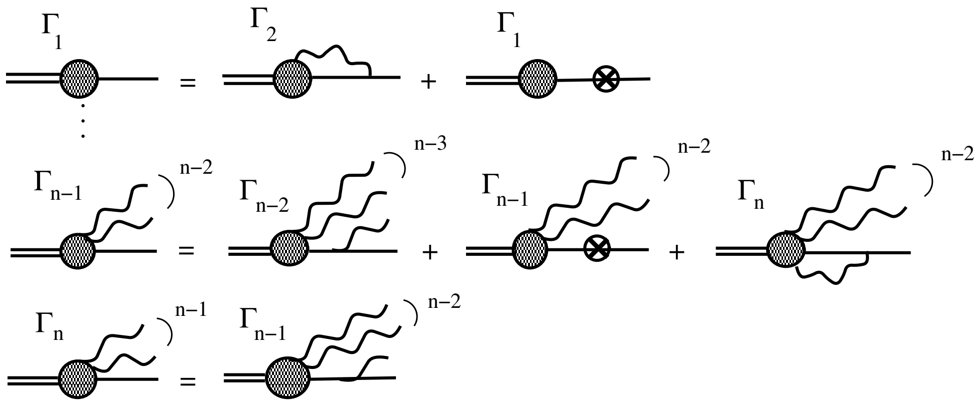
- $m = 0.94$, $\mu = 0.14$
- $\alpha = \frac{g^2}{16\pi m^2}$
- Regularization: $\|\vec{k}\| < L$

$L(\alpha = 3)$	5	10	50	100	200
$\frac{\delta m^2}{\delta m_0^2}$	1.052	1.040	1.025	1.022	1.020
$\langle q \rangle$	0.538	0.565	0.588	0.591	0.593

$\alpha(L = 200)$	1	10	100	1000
$\frac{\delta m^2}{\delta m_0^2}$	1.011	1.029	1.040	1.043

Generalization

Difficulty *in principle* does not grow with n.



$\Rightarrow$ $n \to n-1$ is always trivial

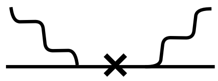
$\Rightarrow$ iteration $i \to i-1$ can be made systematically

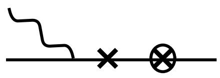
$\Rightarrow$ going from n to $n+1$ requires to restart computation

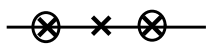
Including spin. . .

Two spinor fields exchanging a scalar: $g\bar{\psi}\varphi\psi$

$\Rightarrow$ Same structure for the 3-body system with 3 additional contact terms:

$g - g$:  $= g^2\left(-\frac{\not\omega}{2\omega.p}\right)$

$g - \delta m$:  $= g\,\delta m\left(-\frac{\not\omega}{2\omega.p}\right)$

$\delta m - \delta m$:  $= \delta m^2\left(-\frac{\not\omega}{2\omega.p}\right)$

$\Rightarrow$ Same difficulty *in principle* as scalar case, with 7 wavefunctions instead of 3

Conclusion

- heavy use of covariant LF special features

- "satisfactory" for 2- and 3-body cases

- iterative method should allow for generalization to n-body case

- computational complexity stays under control, i.e. does not explode for high n

- extended to more physical models...?