Form Factors of Vector Mesons in Light-Front Dynamics

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1. Motivation

The relativistic constituent quark model has had quite some success.

We use it in light-front dynamics (LFD) for the calculation of form factors.

Symmetries limit the form of the currents.

LFD satisfies all symmetries, but not $manifestly$.

Example

Compute electromagnetic properties of a ρ meson using a covariant fermion field theory model in $(3+1)$ dimensions.

Zero-mode shows up in light-front calculation of J^μ.

Avoid zero-mode by choosing

plus-current J^+

valence contribution

helicity components, $(\lambda'\,\lambda) = (++), (+-), (+0)$.

light-front gauge ($\epsilon^+_{\lambda=\pm} = 0$)

BLGB and C.-R. Ji
Frame dependence of spin-one angular conditions in light-front dynamics
Phys. Rev. D **65**, 073002 (2002); hep-ph/019005

BLGB, H.-M. Choi, and C.-R. Ji
The vector meson form-factor analysis in light front dynamics
Phys.Rev.D **65**, 116001 (2002); hep-ph/0202217

2. Spin-1 Current

Symmetries: Hermiticity, Covariance, Time-reversal

Current expressed in terms of real, invariant form factors

$$\begin{aligned}
G^\mu_{h'h} &= \langle p', h' | J^\mu | p, h \rangle \\
&= -\,\epsilon^*_{h'} \cdot \epsilon_h \, (p+p')^\mu \; F_1(Q^2) \\
&\quad + (\epsilon^\mu_h \; q \cdot \epsilon^*_{h'} - \epsilon^{*\mu}_{h'} \; q \cdot \epsilon_h) \;\; F_2(Q^2) \\
&\quad + \frac{(\epsilon^*_{h'} \cdot q)(\epsilon_h \cdot q)}{2m^2} (p+p')^\mu \; F_3(Q^2)
\end{aligned}$$

Physical form factors G_C (charge monopole), G_M (magnetic dipole), and G_Q (electric quadrupole), can be defined as linear combinations of the invariant ones

$$\begin{aligned}
G_C &= \left(1 + \frac{2}{3}\eta\right) \; F_1 + \frac{2}{3}\eta \; F_2 + \frac{2}{3}\eta(1+\eta) \; F_3 \\
G_M &= -F_2 \\
G_Q &= F_1 + F_2 + (1+\eta) \; F_3
\end{aligned}$$

$$\eta = \frac{Q^2}{4m^2}, \quad Q^2 = -(p_2 - p_1)^2$$

LF polarization vectors

$$\left.\begin{matrix} \varepsilon_{\rm LF}(p^+, p^1, p^2; +) \\ \varepsilon_{\rm LF}(p^+, p^1, p^2; 0) \\ \varepsilon_{\rm LF}(p^+, p^1, p^2; -) \end{matrix}\right\} = \left\{ \begin{matrix} \left(0, \frac{-1}{\sqrt{2}}, \frac{-i}{\sqrt{2}}, \frac{p^r}{p^+}\right) \\ \left(\frac{p^+}{m}, \frac{p^1}{m}, \frac{p^2}{m}, \frac{\vec{p}^{\perp\,2} - m^2}{2mp^+}\right) \\ \left(0, \frac{1}{\sqrt{2}}, \frac{-i}{\sqrt{2}}, \frac{p^l}{p^+}\right) \end{matrix}\right.$$

Since we are working only with the plus component of the current, we shall use the following short-hand notations

$$G_a = G^{+}_{++} = G^{+*}_{--}, \quad G_b = G^{+}_{0+} = -G^{+*}_{0-},$$

$$G_c = G^{+}_{+0} = -G^{+*}_{-0}, \; G_d = G^{+}_{00}, \; G_e = G^{+}_{-+} = G^{+*}_{+-}.$$

The matrix elements are related to the form factors as follows

$$\begin{aligned} G_a &= a_1 F_1 + a_3 F_3 \\ G_b &= b_1 F_1 + b_2 F_2 + b_3 F_3 \\ G_c &= c_1 F_1 + c_2 F_2 + c_3 F_3 \\ G_d &= d_1 F_1 + d_2 F_2 + d_3 F_3 \\ G_e &= e_3 F_3 \end{aligned}$$

The coefficients $a_1, \ldots e_3$ depend on the *kinematics*.

3. Angular Conditions

Extract invariant form factors from current matrix elements

Simplest scheme:

Solve first for F_3 from G_e;

F_1 is obtained from G_a and F_3;

Three options for obtaining F_2 from G_b, G_c, or G_d

denoted as F_2^b, F_2^c, and F_2^d, respectively.

$$\begin{aligned}
F_1 &= \frac{1}{a_1} G_a - \frac{a_3}{a_1 e_3} G_e \\
F_3 &= \frac{1}{e_3} G_e \\
F_2^b &= \frac{1}{b_2}\left[-\frac{b_1}{a_1} G_a + G_b + \frac{a_3 b_1 - a_1 b_3}{a_1 e_3} G_e \right] \\
F_2^c &= \frac{1}{c_2}\left[-\frac{c_1}{a_1} G_a + G_c + \frac{a_3 c_1 - a_1 c_3}{a_1 e_3} G_e \right] \\
F_2^d &= \frac{1}{d_2}\left[-\frac{d_1}{a_1} G_a + G_d + \frac{a_3 d_1 - a_1 d_3}{a_1 e_3} G_e \right]
\end{aligned}$$

Two additional constraints on the current operator are required.

These consistency conditions are the angular conditions

$$\Delta_{\mathrm{bc}} = F_2^b - F_2^c, \quad \Delta_{\mathrm{bd}} = F_2^b - F_2^d$$

One can obtain the following helicity amplitudes in the $q^+ = 0$ frame

$$
\begin{aligned}
G_a &= 2p^+(F_1 + \eta F_3) \\
G_c &= p^+\sqrt{2\eta}(2F_1 + F_2 + 2\eta F_3) \\
G_e &= -2p^+\eta F_3 \\
G_d &= 2p^+\left[(1-2\eta)F_1 - 2\eta F_2 - 2\eta^2 F_3\right]
\end{aligned}
$$

Usually, only one angular condition relating the four helicity amplitudes is given

$$
\Delta(Q^2) = (1+2\eta)G_a + G_e - \sqrt{8\eta}G_c - G_d = 0
$$

Choose which matrix elements to use to extract the form factors

Grach and Kondratyuk, Sov. J. Nucl. Phys. 39, 198 (1984)

$$
\begin{aligned}
G_C^{\mathrm{GK}} &= \frac{1}{2p^+}\left[\frac{(3-2\eta)}{3}G_a + \frac{4\eta}{3}\frac{G_c}{\sqrt{2\eta}} + \frac{1}{3}G_e\right] \\
G_M^{\mathrm{GK}} &= \frac{2}{2p^+}\left[G_a - \frac{1}{\sqrt{2\eta}}G_c\right] \\
G_Q^{\mathrm{GK}} &= \frac{1}{2p^+}\left[-G_a + 2\frac{G_c}{\sqrt{2\eta}} - \frac{G_e}{\eta}\right]
\end{aligned}
$$

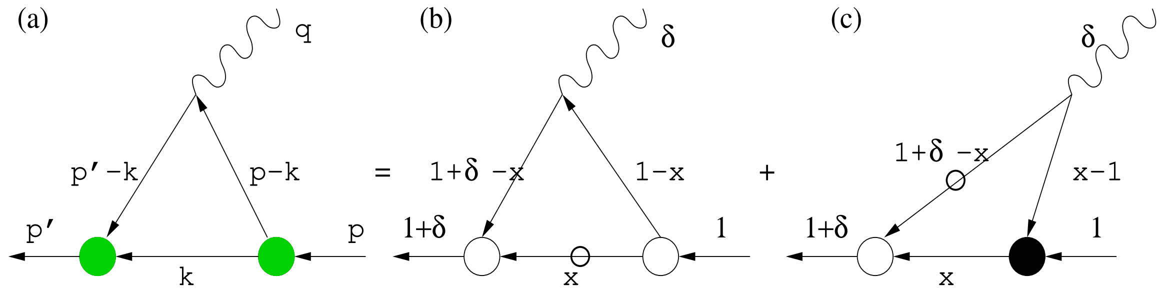
Chung, Coester, Keister, and Polyzou, Phys. Rev. C 37, 2000 (1988)

$$
\begin{aligned}
G_C^{\mathrm{CCKP}} &= \frac{1}{2p^+(1+\eta)}\left[\frac{3-2\eta}{6}(G_a + G_d) + \frac{10\eta}{3}\frac{G_c}{\sqrt{2\eta}} + \frac{4\eta-1}{6}G_e\right] \\
G_M^{\mathrm{CCKP}} &= \frac{1}{2p^+(1+\eta)}\left[G_a + G_d - G_e - \frac{2(1-\eta)}{\sqrt{2\eta}}G_c\right] \\
G_Q^{\mathrm{CCKP}} &= \frac{1}{2p^+(1+\eta)}\left[-\frac{1}{2}(G_a + G_d) + 2\frac{G_c}{\sqrt{2\eta}} - \frac{(\eta+2)}{2\eta}G_e\right]
\end{aligned}
$$

Brodsky and Hiller, Phys. Rev. D 46, 2141 (1992)

$$
\begin{aligned}
G_C^{\mathrm{BH}} &= \frac{1}{2p^+(1+2\eta)}\left[\frac{(3-2\eta)}{3}G_d + \frac{16\eta}{3}\frac{G_c}{\sqrt{2\eta}} + \frac{2(2\eta-1)}{3}G_e\right] \\
G_M^{\mathrm{BH}} &= \frac{2}{2p^+(1+2\eta)}\left[G_d - G_e + \frac{(2\eta-1)}{\sqrt{2\eta}}G_c\right] \\
G_Q^{\mathrm{BH}} &= \frac{1}{2p^+(1+2\eta)}\left[2\frac{G_c}{\sqrt{2\eta}} - G_d - \frac{(1+\eta)}{\eta}G_e\right].
\end{aligned}
$$

4. Model



(a) covariant triangle diagram

(b) LF valence diagram defined in $0 < k^+ < p^+$

(c) LF nonvalence diagram defined in $p^+ < k^+ < p'^+$

$\delta = q^+/p^+ = p'^+/p^+ - 1.$

LF wave-function vertex (open circle)

Non-wave-function vertex (filled circle)

The small circles in (b) and (c) represent the on-shell pole

of the quark propagator determined from the k^--integration.

Full covariant amplitude is given by

$$
\begin{aligned}
G^{\mu}_{h'h}(0) &= iN_c g^2 \int \frac{d^4k}{(2\pi)^4} \frac{S_\Lambda(k-p) S^{\mu}_{h'h} S_\Lambda(k-p')}{D_1 D_2 D_3}, \\
(S^{+}_{h'h}) &= \mathrm{Tr}[\not\epsilon^{*}_{h'}(\not k - \not p' + m_q)\gamma^{+}(\not k - \not p + m_q)\ \not\epsilon_h(\not k + m_q)] \\
D_1 &= [(k-p)^2 - m_q^2 + i\varepsilon] \\
D_2 &= [k^2 - m_q^2 + i\varepsilon] \\
D_3 &= [(k-p')^2 - m_q^2 + i\varepsilon]
\end{aligned}
$$

EM vertex smearing

$$S_\Lambda(p) = \Lambda^2/(p^2 - \Lambda^2 + i\epsilon)$$

Parameters:
$M_\rho = 0.770$ GeV/c^2, $m_q = 0.430$ GeV/c^2, $\Lambda = 1.60$ GeV/c^2

Zero mode

Fermion propagator has LF propagating part and instantaneous part

$$\not{P} + m_q = (\not{P}_{\rm on} + m_q) + \gamma^+(P^- - P^-_{\rm on})$$

Corresponding split of the trace

$$S^+_{h'h}(k-p, k-p', k) = (S^+_{h'h})_{\rm on} + (S^+_{h'h})_{\rm inst}$$

If the integration range of the nonvalence region, $p^+ < k^+ < p^+ + q^+$, shrinks to zero in the $q^+ \to 0$ limit, the nonvalence contribution may not vanish, but give a finite contribution,

$$\lim_{q^+\to 0} \int_{p^+}^{p^+ + q^+} dk^+ (\cdots) \equiv \lim_{\delta\to 0} \int_1^{1+\delta} dx (\cdots) \neq 0.$$

It is called the *zero-mode* contribution.

5. Results

Kinematics

Three different kinematics were used, (p^+, p_x, p_y, p^-)

Drell-Yan-West

$$\begin{aligned} p &= (p^+, 0, 0, m^2/(2p^+)) \\ p' &= (p^+, Q\cos\phi, Q\sin\phi, (Q^2+m^2)/(2p^+)) \end{aligned}$$

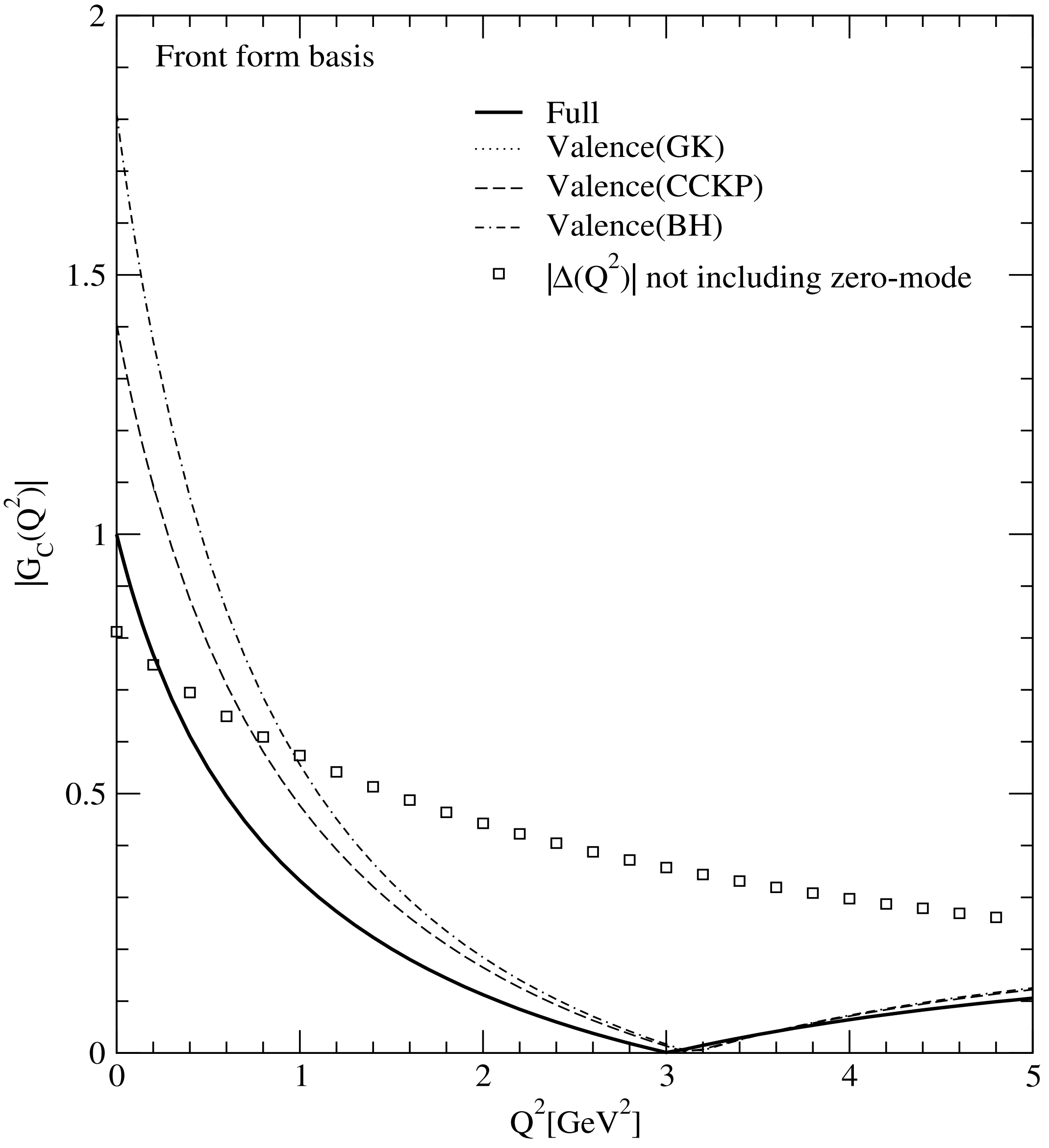
Breit frame

$$\begin{aligned} \beta &= \sqrt{1+\left(\frac{Q}{2m}\right)^2} \\ p &= \left(\frac{2m\beta - Q\cos\theta}{2\sqrt{2}}, -\frac{Q\sin\theta\cos\phi}{2}, -\frac{Q\sin\theta\sin\phi}{2}, \frac{2m\beta + Q\cos\theta}{2\sqrt{2}}\right) \\ p' &= \left(\frac{2m\beta + Q\cos\theta}{2\sqrt{2}}, \frac{Q\sin\theta\cos\phi}{2}, \frac{Q\sin\theta\sin\phi}{2}, \frac{2m\beta - Q\cos\theta}{2\sqrt{2}}\right) \end{aligned}$$

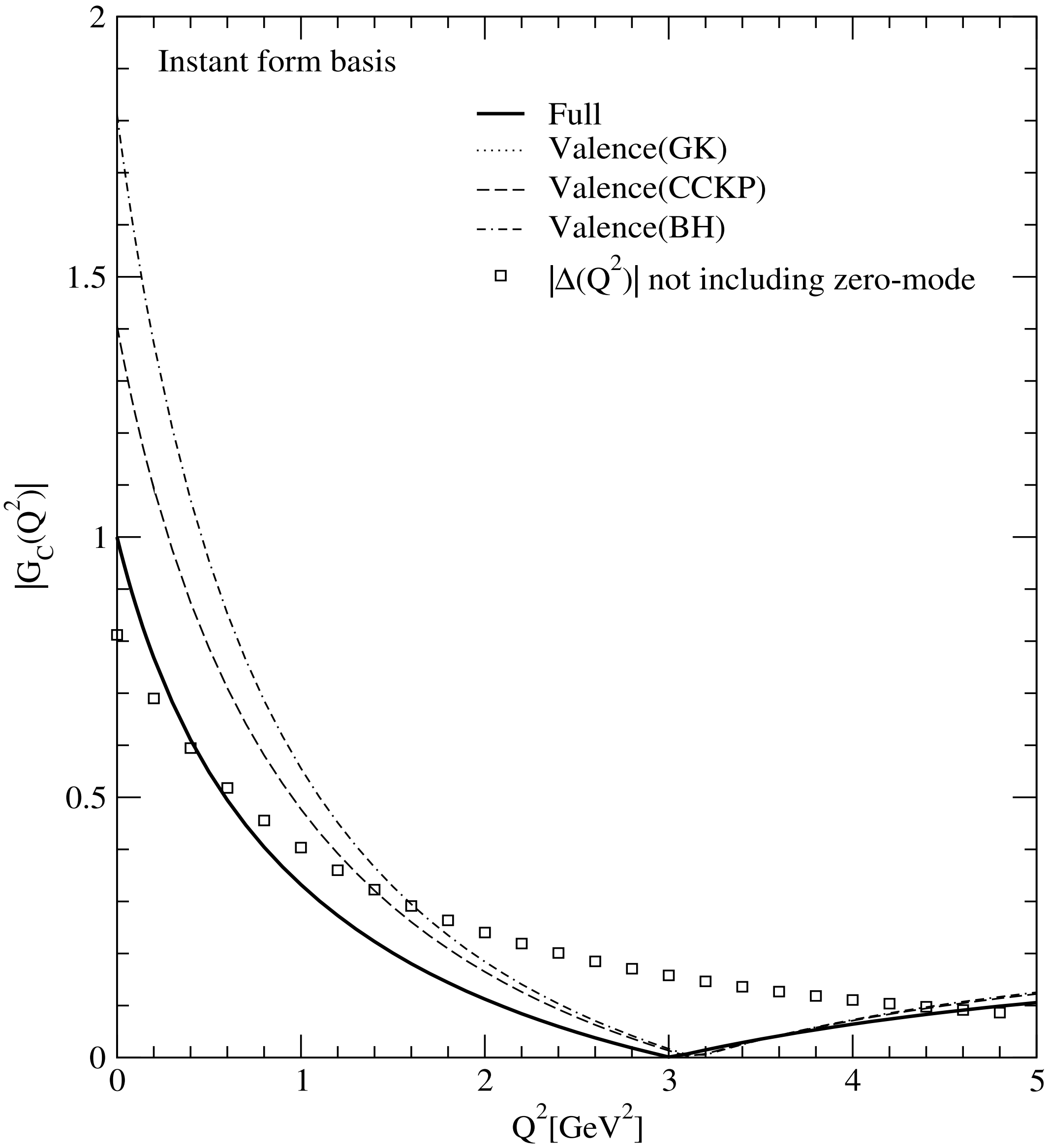
Target rest frame

$$\begin{aligned} \kappa &= \frac{Q^2}{2m} \\ p &= \left(\frac{m}{\sqrt{2}}, 0, 0, \frac{m}{\sqrt{2}}\right) \\ p' &= \left(\frac{m+\kappa+\beta Q\cos\theta}{\sqrt{2}}, \beta Q\sin\theta\cos\phi, \beta Q\sin\theta\sin\phi, \frac{m+\kappa-\beta Q\cos\theta}{\sqrt{2}}\right) \end{aligned}$$

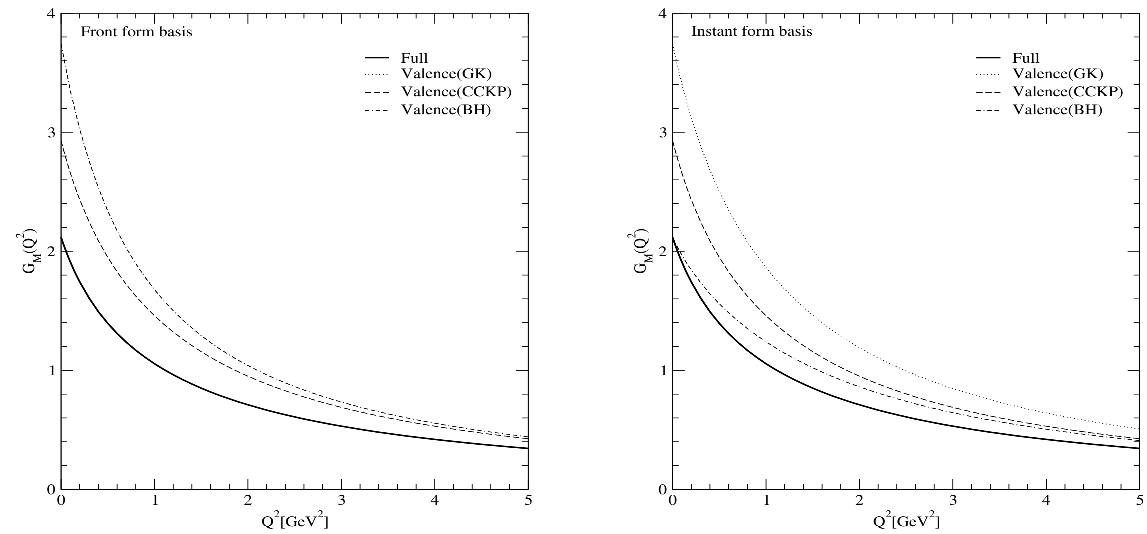
Charge form factor $|G_C(Q^2)|$. Front-form basis



Charge form factor $|G_C(Q^2)|$. Instant-form basis

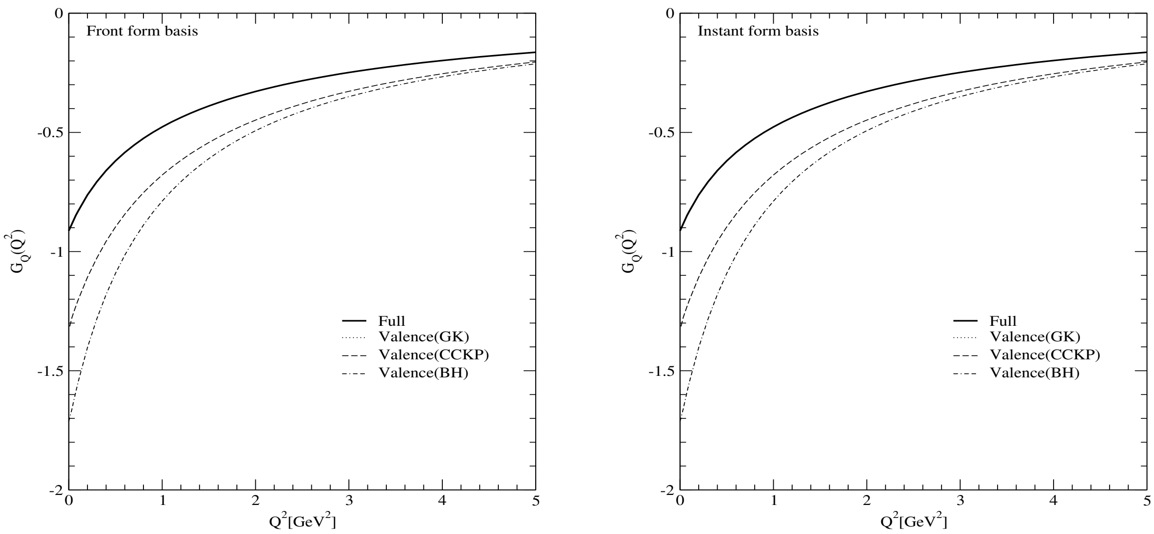


Magnetic form factor $|G_M(Q^2)|$



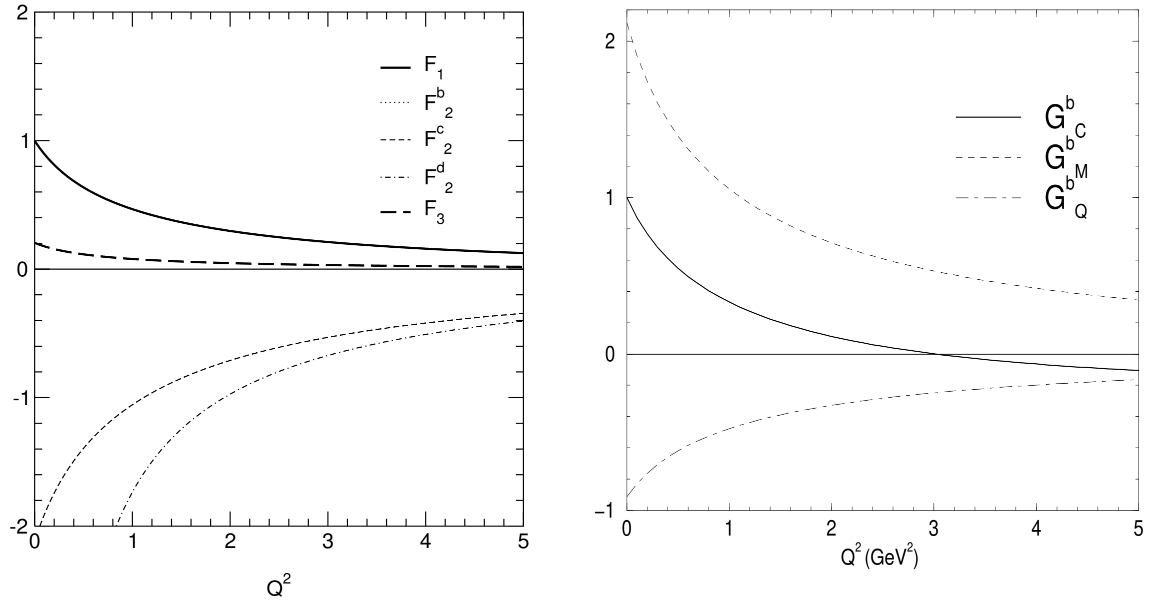
$G_M(Q^2)$ obtained from the light-front (left) and the instant form (right) polarization vectors

Quadrupole form factor $|G_Q(Q^2)|$

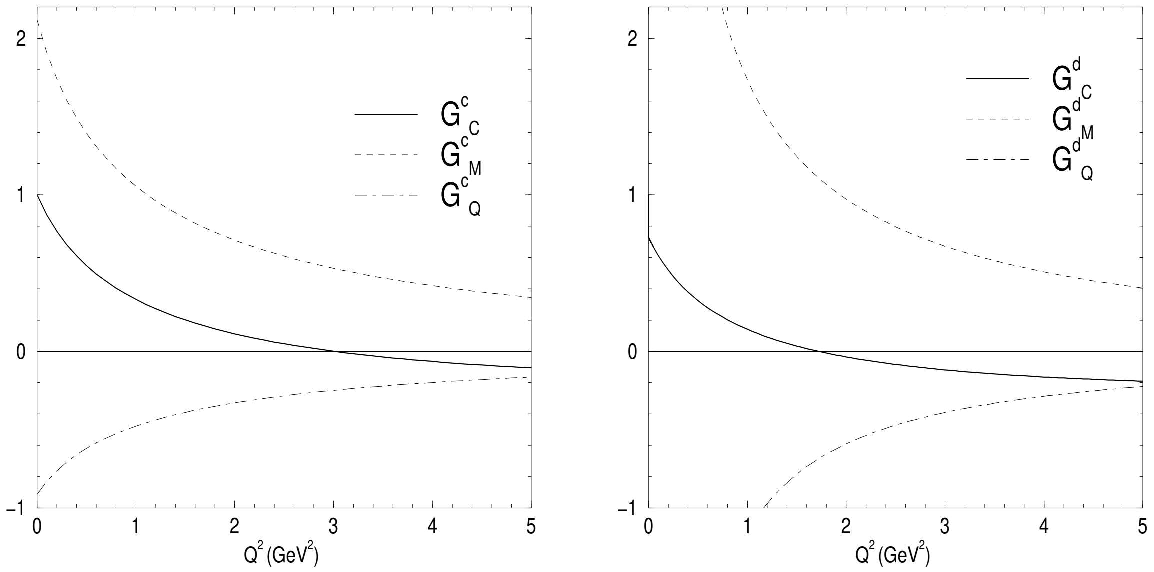


$G_Q(Q^2)$ obtained from the light-front (left) and the instant form (right) polarization vectors

Drell-Yan-West frame. Valence parts

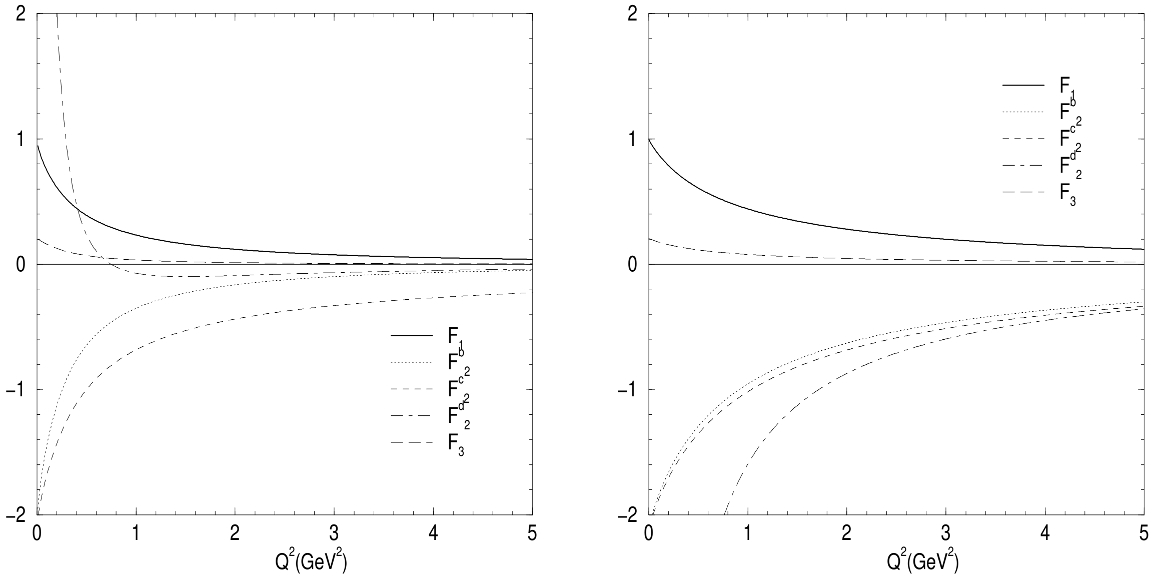


F_1, F_2^b, F_2^c, F_2^d, and F_3 (left) and G_C, G_M, and G_Q from F_2^b (right)



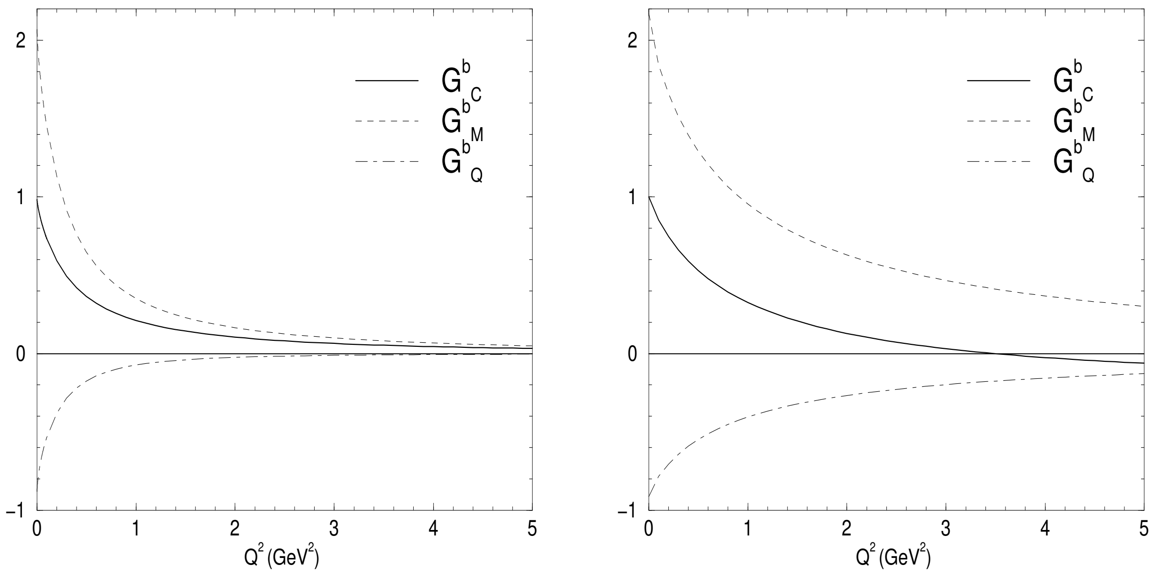
G_C, G_M, and G_Q from F_2^c (left) and F_2^d (right)

Breit frame. Invariant form factors. Valence parts

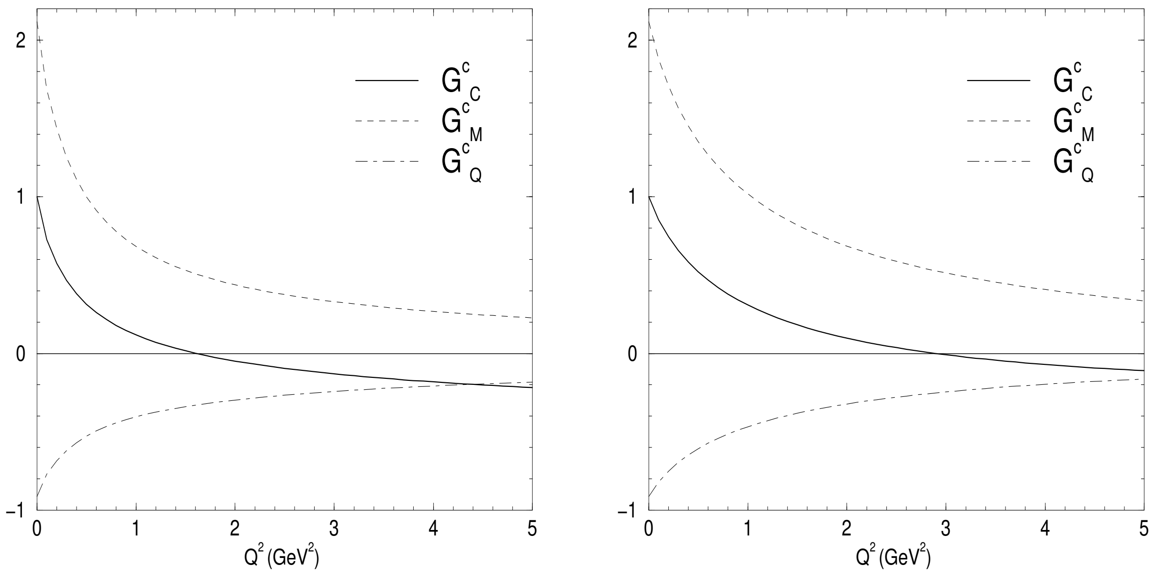


F_1, F_2^b, F_2^c, F_2^d, and F_3 for $\theta = \pi/20$ (left) and $\theta = 9\pi/20$ (right)

Breit frame. Physical form factors. Valence parts

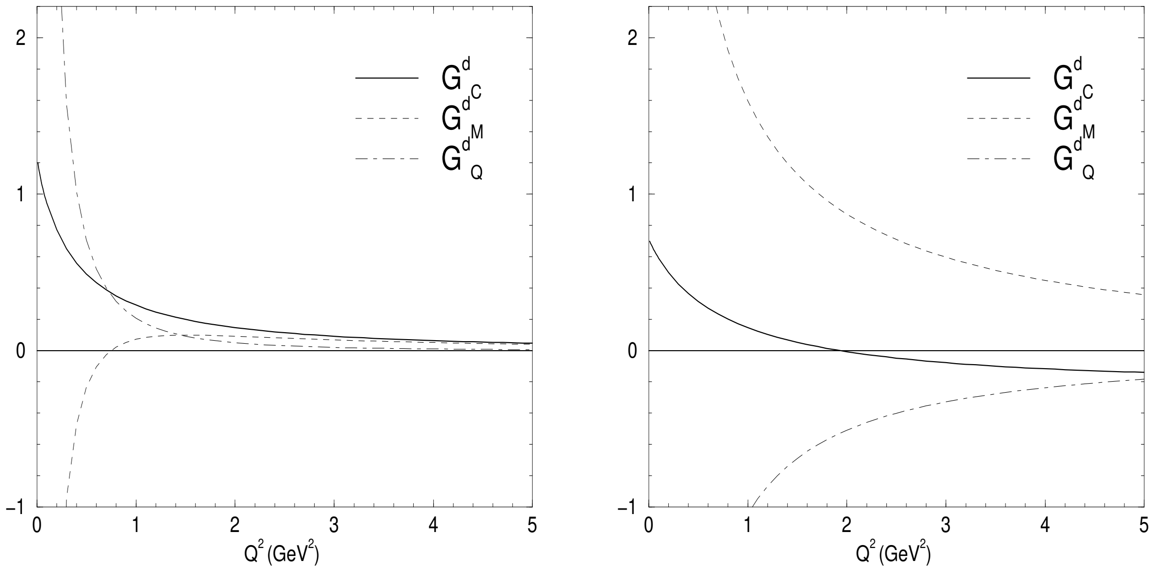


G_C, G_M, and G_Q from F_2^b, $\theta = \pi/20$ (left), $\theta = 9\pi/20$ (right)



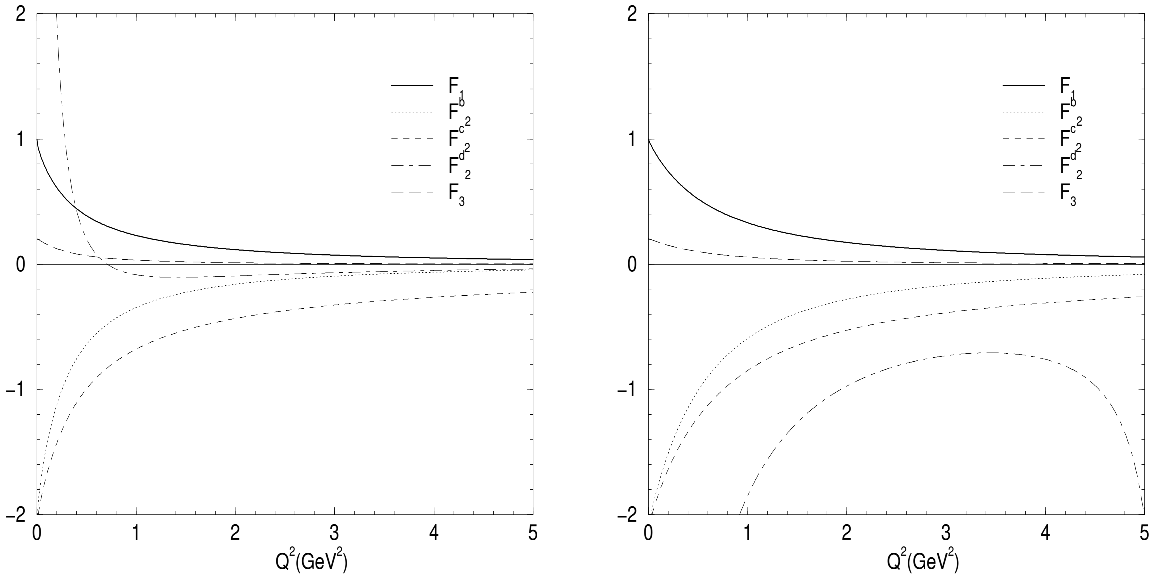
G_C, G_M, and G_Q from F_2^c, $\theta = \pi/20$ (left), $\theta = 9\pi/20$ (right)

Breit frame. Physical form factors. Valence parts



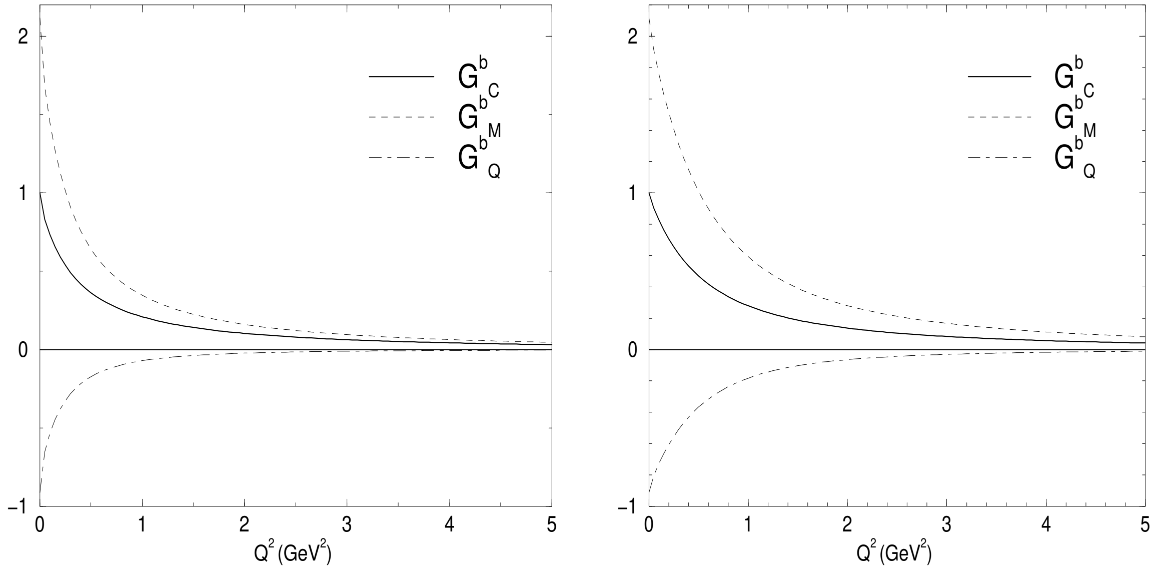
G_C, G_M, and G_Q from F_2^d, $\theta = \pi/20$ (left), $\theta = 9\pi/20$ (right)

Target-rest frame. Invariant form factors. Valence parts

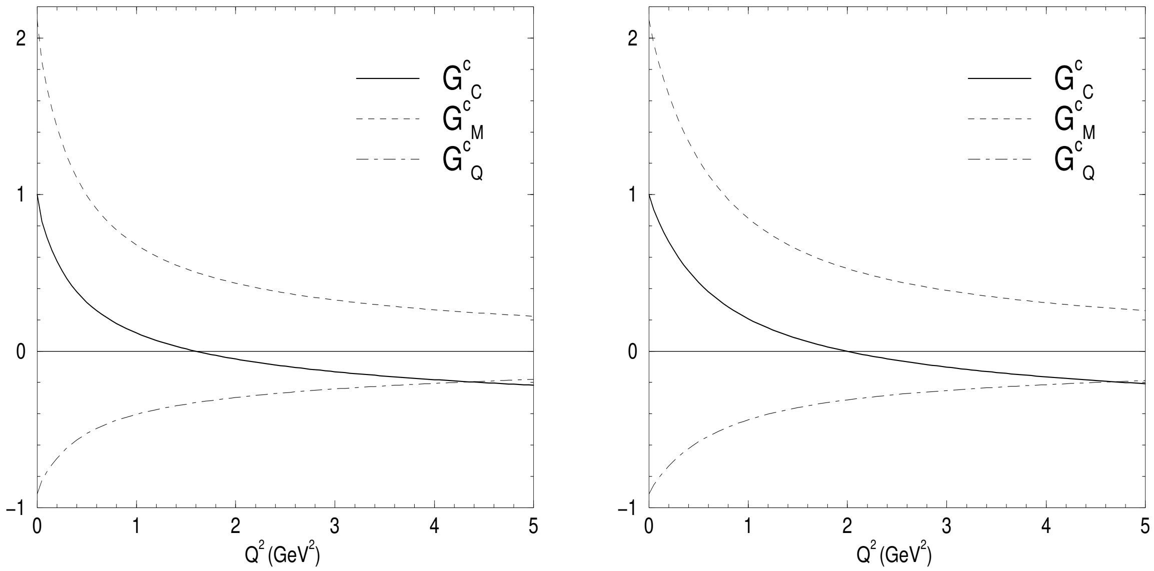


F_1, F_2^b, F_2^c, F_2^d, and F_3 for $\theta = \pi/20$ (left) and $\theta = 9\pi/20$ (right)

Target-rest frame. Physical form factors. Valence parts

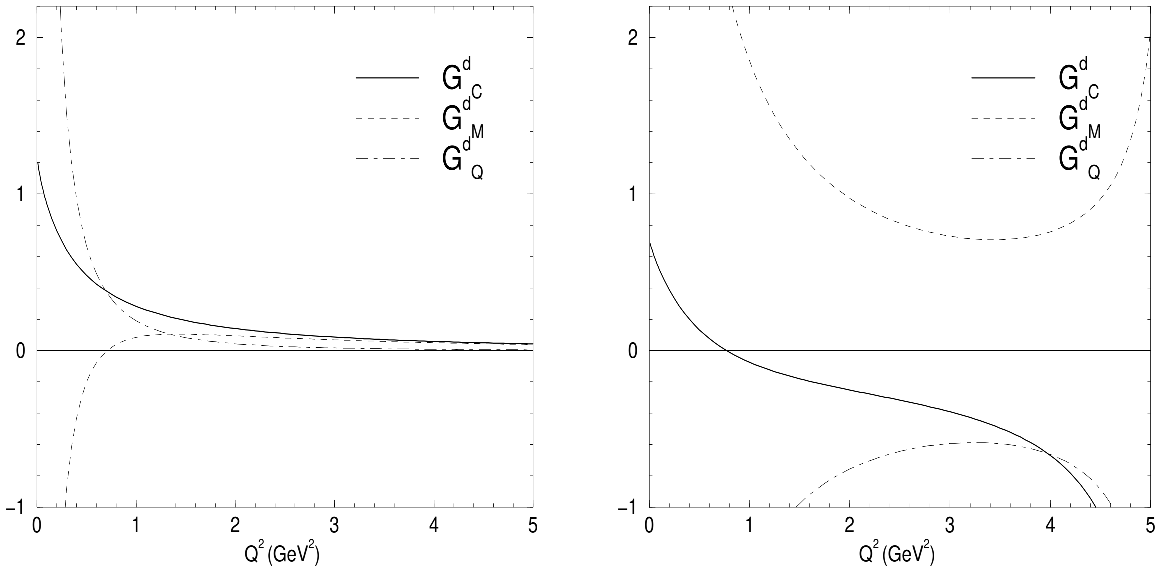


G_C, G_M, and G_Q from F_2^b, $\theta = \pi/20$ (left), $\theta = 9\pi/20$ (right)



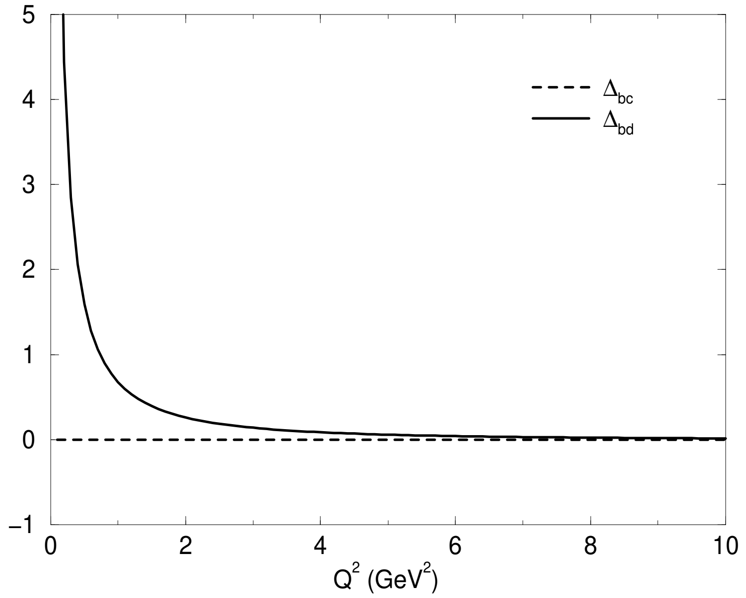
G_C, G_M, and G_Q from F_2^c, $\theta = \pi/20$ (left), $\theta = 9\pi/20$ (right)

Target-rest frame. Physical form factors. Valence parts



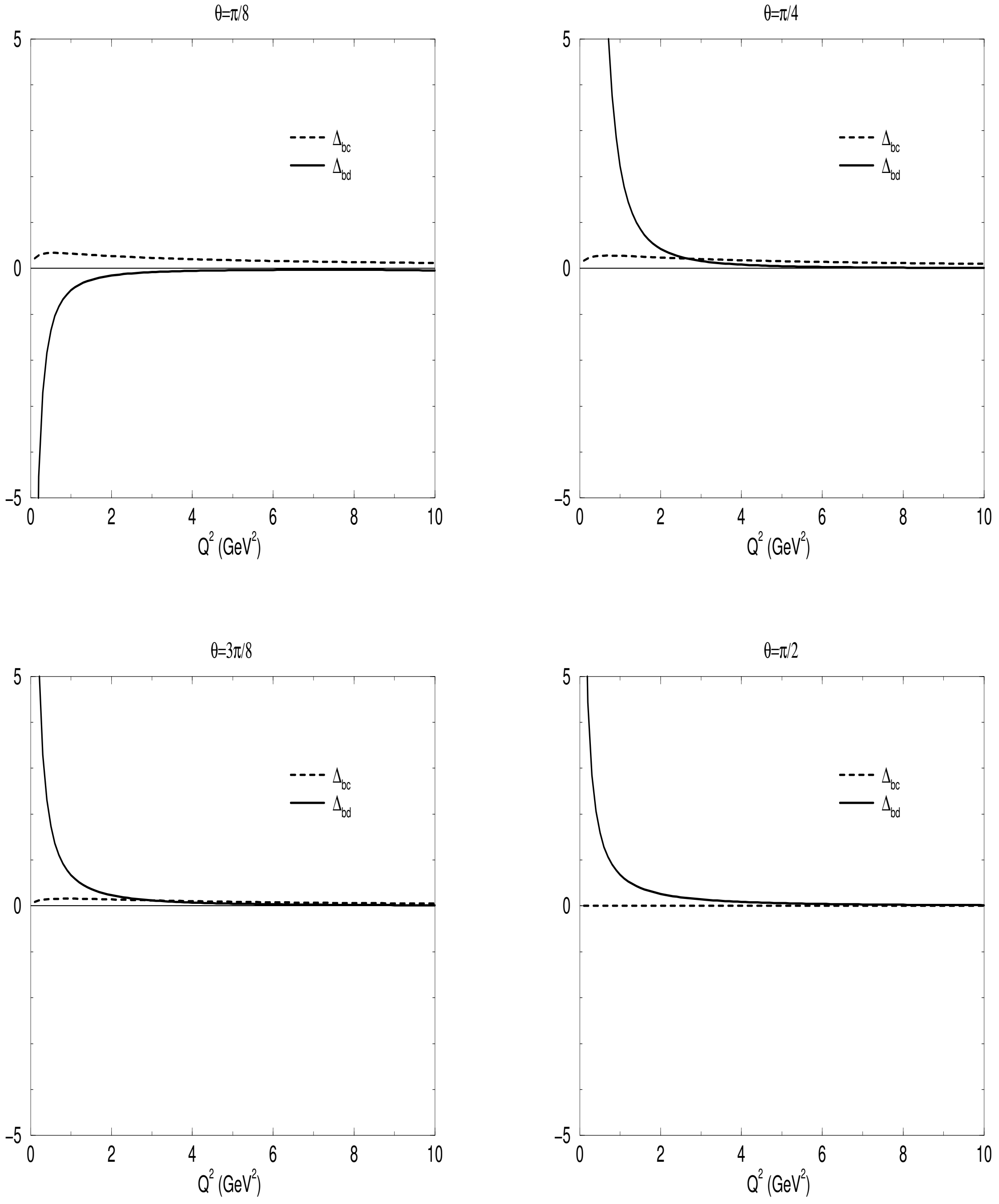
G_C, G_M, and G_Q from F_2^d, $\theta = \pi/20$ (left), $\theta = 9\pi/20$ (right)

Drell-Yan-West frame. Angular conditions



Note that for asymptotic values of Q^2 the second angular condition $\Delta_{\rm bd} = 0$ is also fulfilled.

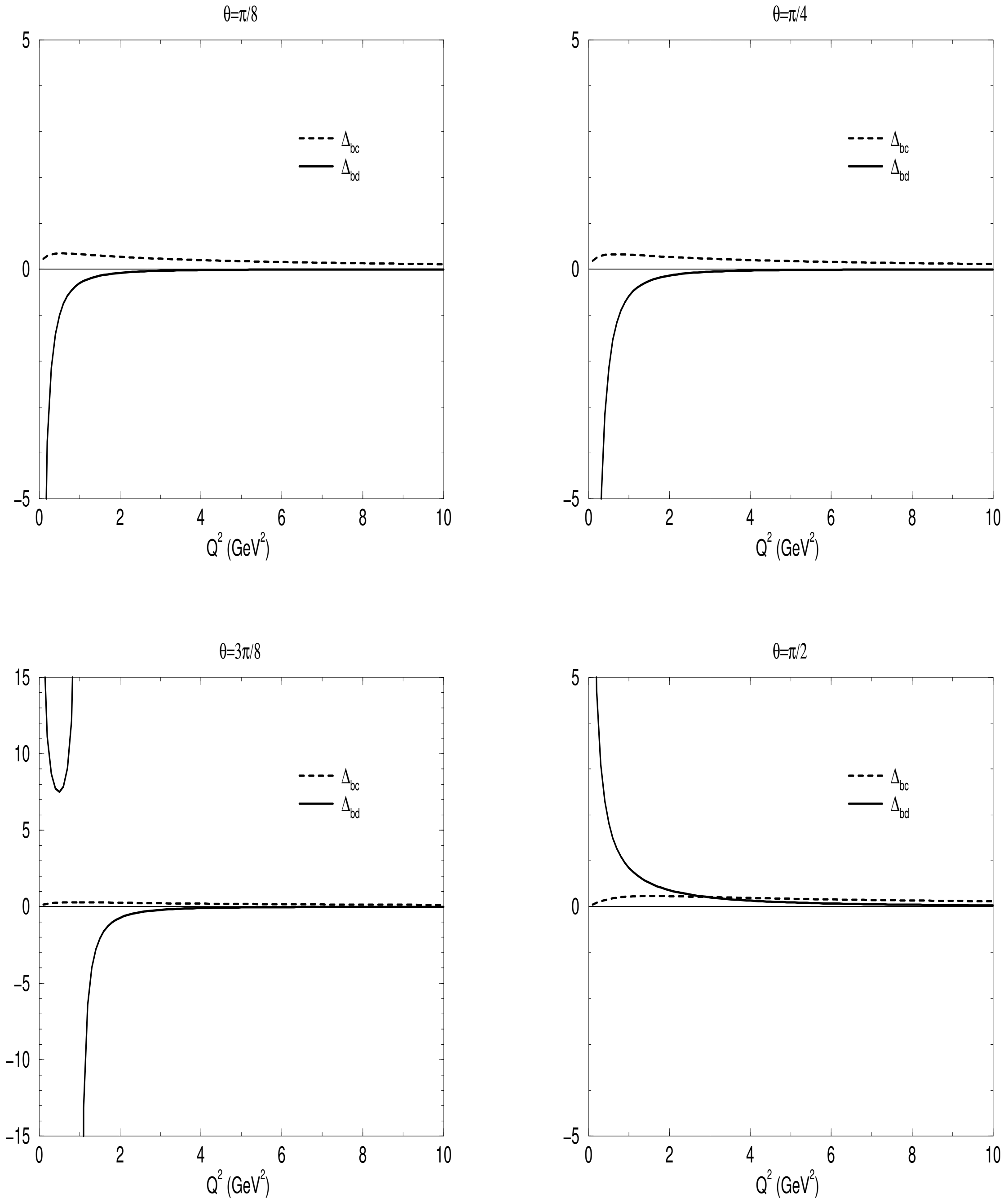
Breit frame. Angular conditions



Dependence of the angular conditions $\Delta_{\rm bc}$ and $\Delta_{\rm bd}$ on Q^2 for four different angles θ

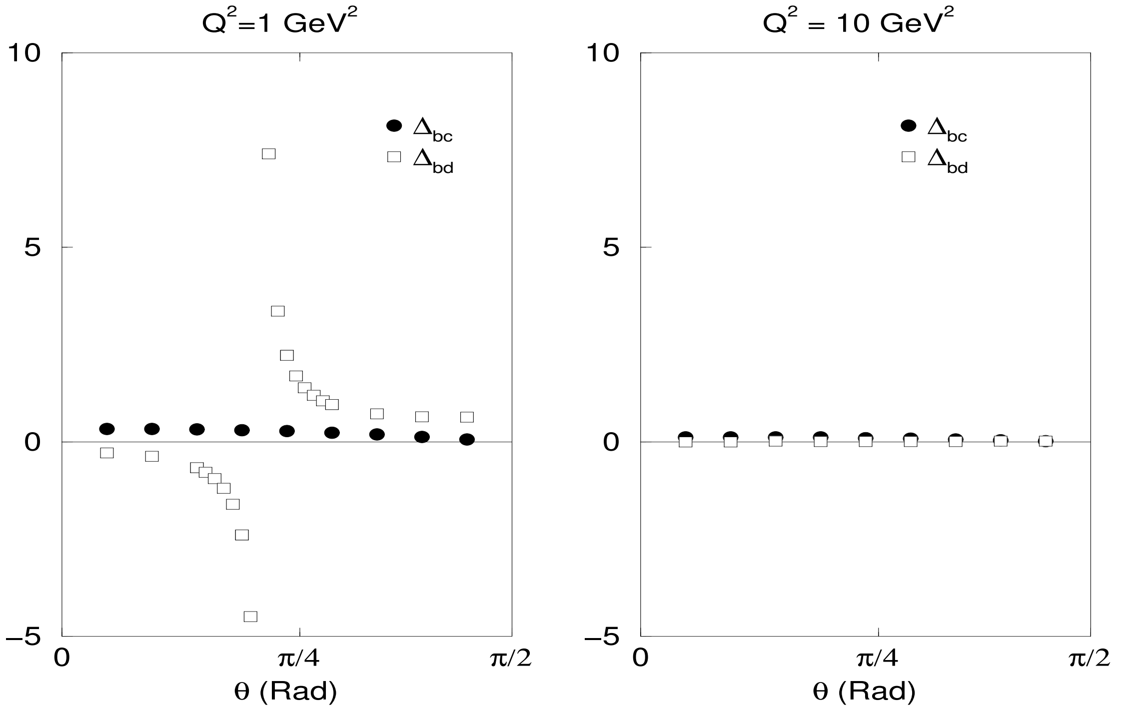
Note: The quantities $\Delta_{\rm bc}$ and $\Delta_{\rm bd}$ for $\theta = \pi/2$ are identical to the ones for the DYW frame.

Target-rest frame. Angular conditions

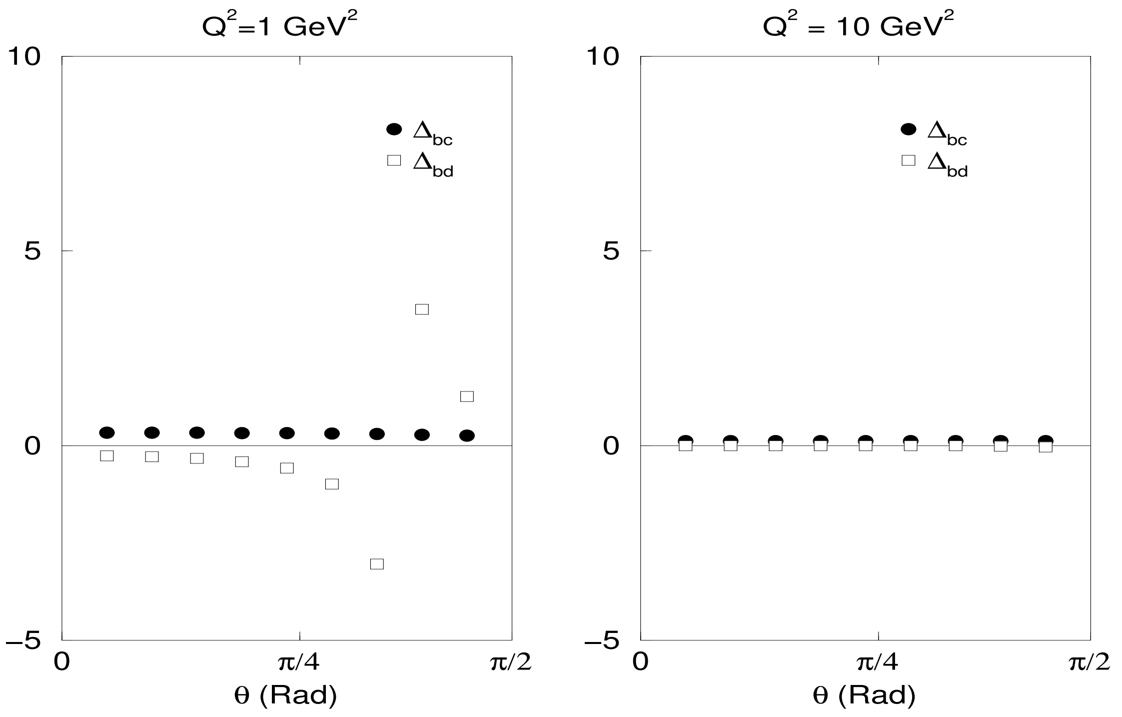


Dependence of the angular conditions $\Delta_{\rm bc}$ and $\Delta_{\rm bd}$ on Q^2 for four different angles θ

Breit frame. Angular conditions

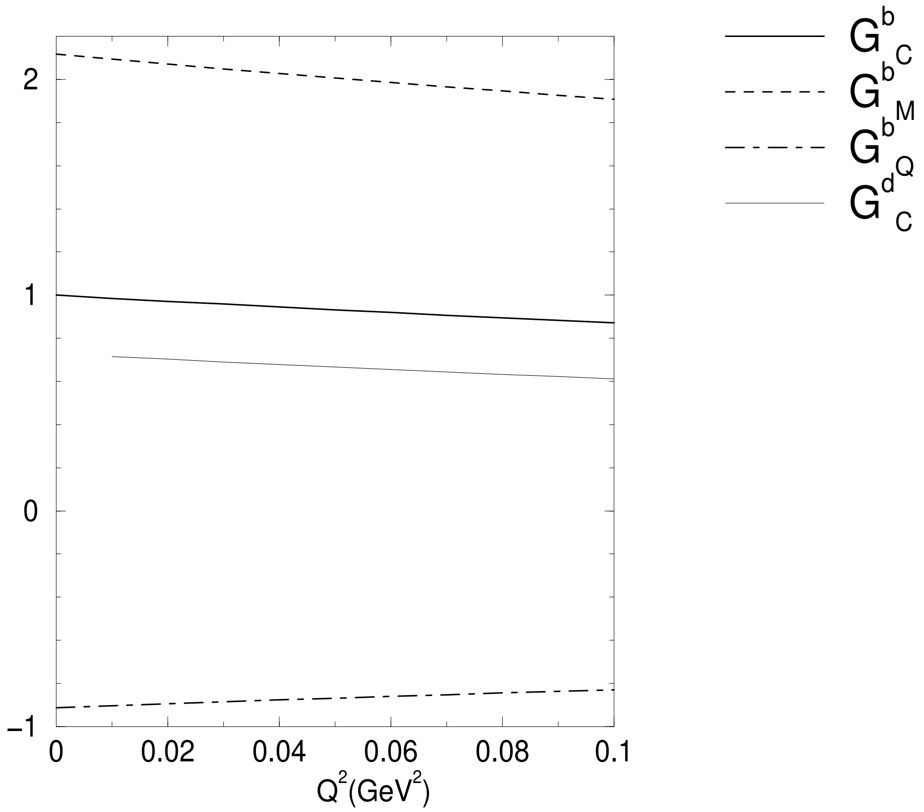


Target-rest frame. Angular conditions



$\Delta_{\rm bc}$ and $\Delta_{\rm bd}$ for $Q^2 = 1.0$ and 10.0 GeV^2 for different angles θ.

Radii



Valence contributions to physical form factors for small values of Q^2 in DYW frame. While b and d variants are shown here, the d-variant magnetic(G^d_M) and quadrupole(G^d_Q) form factors are out of scale because they diverge as $Q^2 \to 0$.

Linear fit is used to determine the radii

$$\begin{aligned}
G_C(Q^2) &\sim G_C(0)\left[1-\frac{1}{6}\langle r_C^2\rangle Q^2\right], \\
G_M(Q^2) &\sim G_M(0)\left[1-\frac{1}{10}\langle r_M^2\rangle Q^2\right], \\
G_Q(Q^2) &\sim G_Q(0)\left[1-\frac{1}{14}\langle r_Q^2\rangle Q^2\right].
\end{aligned}$$

Squared radii in GeV^{-2} for the different reference frames and variants.

Ref. frame, variant	G_C	G_M	G_Q
DYW, G^b and G^c	7.63	9.73	12.6
DYW, G^d	9.56	div.	div.
	$\theta = \pi/20$		
BRT, tot	7.63	9.73	12.6
BRT, G^b, val	14.3	27.6	50.4
BRT, G^c, val	14.7	18.3	19.4
BRT, G^d, val	13.9	div.	div.
	$\theta = 9\pi/20$		
BRT, tot	7.63	9.73	12.6
BRT, G^b, val	8.42	12.0	17.7
BRT, G^c, val	8.48	10.7	13.6
BRT, G^d, val	10.0	div.	div.
	$\theta = \pi/20$		
TRF, tot	7.63	9.73	12.6
TRF, G^b, val	14.3	27.7	50.6
TRF, G^c, val	14.8	18.3	19.5
TRF, G^d, val	14.1	div.	div.
	$\theta = 9\pi/20$		
TRF, tot	7.63	9.73	12.6
TRF, G^b, val	9.84	15.9	26.2
TRF, G^c, val	9.98	12.5	15.2
TRF, G^d, val	15.5	div.	div.

6. Conclusions

Constituent model can be used

only valence parts are included

Current

Plus component J^+ must be used

Gauge

LF gauge $\varepsilon^+_{\rm LF}(h = \pm) = 0$ must be used

Reference frame

Frames with $q^+ = 0$ must be used, $i.e.,$

Drell-Yan-West frame or one that is kinematically connected

Extraction scheme

b- or c-variant must be used

Grach-Kondratyuk scheme

Errors

If these four conditions are not fulfilled, large errors may occur

- radii
- form factors

Asymptotically angular conditions satisfied