Simulation of Supersymmetric Chern–Simons Theories [a]

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[a] Work in collaboration with John Hiller and Steve Pinsky.

Introduction/Motivation

Pure SYM theories are known to be “stringy”;
low lying states have more and more partons:

- phenomenologically not interesting
- numerically difficult

Chern-Simons (CS) term gives a SUSY effective mass to the partons

- theory has “QCD-like” light states
- numerically well behaved

Dimensional reduction to 1+1 dimensions

- need to establish benchmarks and tests
- numerically simpler
- 2D results are related non-trivially to 3D results

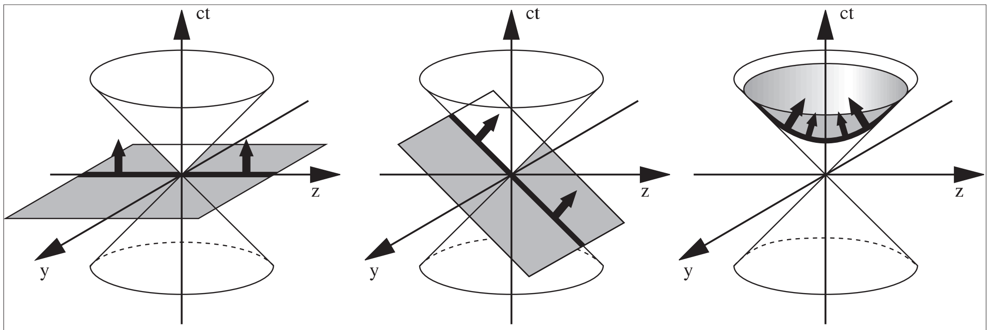
2+1 dimensions

- search for approximate BPS states as in 1+1
- full 3D structure functions
- anomalously light states

Method: SDLCQ

(Supersymmetric Discretized Light-Cone Quantization)

A discrete, Hamiltonian, manifestly supersymm. approach to solving QFTs.



The instant form	The front form	The point form
$\tilde{x}^0 = ct$	$\tilde{x}^0 = ct+z$	$\tilde{x}^0 = \tau \;,\; ct = \tau\cosh\omega$
$\tilde{x}^1 = x$	$\tilde{x}^1 = x$	$\tilde{x}^1 = \omega \;,\; x = \tau\sinh\omega\sin\theta\cos\phi$
$\tilde{x}^2 = y$	$\tilde{x}^2 = y$	$\tilde{x}^2 = \theta \;,\; y = \tau\sinh\omega\sin\theta\sin\phi$
$\tilde{x}^3 = z$	$\tilde{x}^3 = ct-z$	$\tilde{x}^3 = \phi \;,\; x = \tau\sinh\omega\cos\theta$

$$\tilde{g}_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \qquad \tilde{g}_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{2} \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ \frac{1}{2} & 0 & 0 & 0 \end{pmatrix} \qquad \tilde{g}_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -\tau^2 & 0 & 0 \\ 0 & 0 & -\tau^2\sinh^2\omega & 0 \\ 0 & 0 & 0 & -\tau^2\sinh^2\omega\sin^2\theta \end{pmatrix}$$

Light-cone coordinates : $(x^+, x^-, \vec{x}^\perp)$

$x^\pm = (x^0 \pm x^1)/\sqrt{2}$ where x^+/x^- is time/space coordinate

$(P^+)p^+$ (total) longitudinal momentum

P^- light-cone energy (Hamiltonian)

Discretization: in Light-Cone Quantization: $p_i^+ \geq 0$

$$p_i^+ = \frac{n_i P^+}{K}; \qquad n = 1, 2, 3, \ldots, K$$

$K \equiv P^+ \frac{L}{\pi}$ is harmonic resolution (maximal number of partons)

Continuum limit: $K \to \infty$.

SUSY: Preserve SUSY even at finite cutoff by *discretizing first* the supercharge Q^- and *then constructing* the Hamiltonian

$$P^- = \frac{1}{2\sqrt{2}}\{Q^-, Q^-\}.$$

Supersymmetric Chern-Simons gauge theory in $D = 2+1$

Action

$$S = \int d^3x \left(-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + i\bar{\Psi} D_\mu \gamma^\mu \Psi + \frac{\kappa}{2} \mathcal{L}_{\rm CS} \right).$$

where

$$\mathcal{L}_{\rm CS} = \epsilon^{\mu\nu\lambda} \left(A_\mu \partial_\nu A_\lambda + \frac{2i}{3} g \, A_\mu A_\nu A_\lambda \right) + 2\bar{\Psi}\Psi \, .$$

$$\Psi = 2^{-1/4} (\psi, \chi)^t, \qquad D_\mu \Psi = \partial_\mu \Psi + ig[A_\mu, \Psi],$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + ig[A_\mu, A_\nu].$$

All fields in adjoint representation, work in the large N_c limit.

CS supercharge in Fock space

$$Q^-_{CS} = 2^{-1/4} \sqrt{\frac{L}{\pi}} \kappa \sum_n \frac{1}{\sqrt{n}} \left(A^\dagger(n) B(n) + B^\dagger(n) A(n) \right)$$

Note: The CS coupling behaves very much like a complex transverse momentum!

Cf. the similar structure of the transverse momentum component of the supercharge in the full 2+1 dimensional theory:

$$Q^-_\perp = i 2^{7/4} \frac{\sqrt{\pi L}}{L_\perp} \sum_{n,n_\perp} \frac{n_\perp}{\sqrt{n}} \left[A^\dagger(n,n_\perp) B(n,n_\perp) - B^\dagger(n,n_\perp) A(n,n_\perp) \right],$$

the square ($\sim$ Hamiltonian) of which behaves like a part of the light-cone kinetic energy: $(m^2 + k_\perp^2)/k^+$.

SYM-CS(1+1): “BPS-like” states

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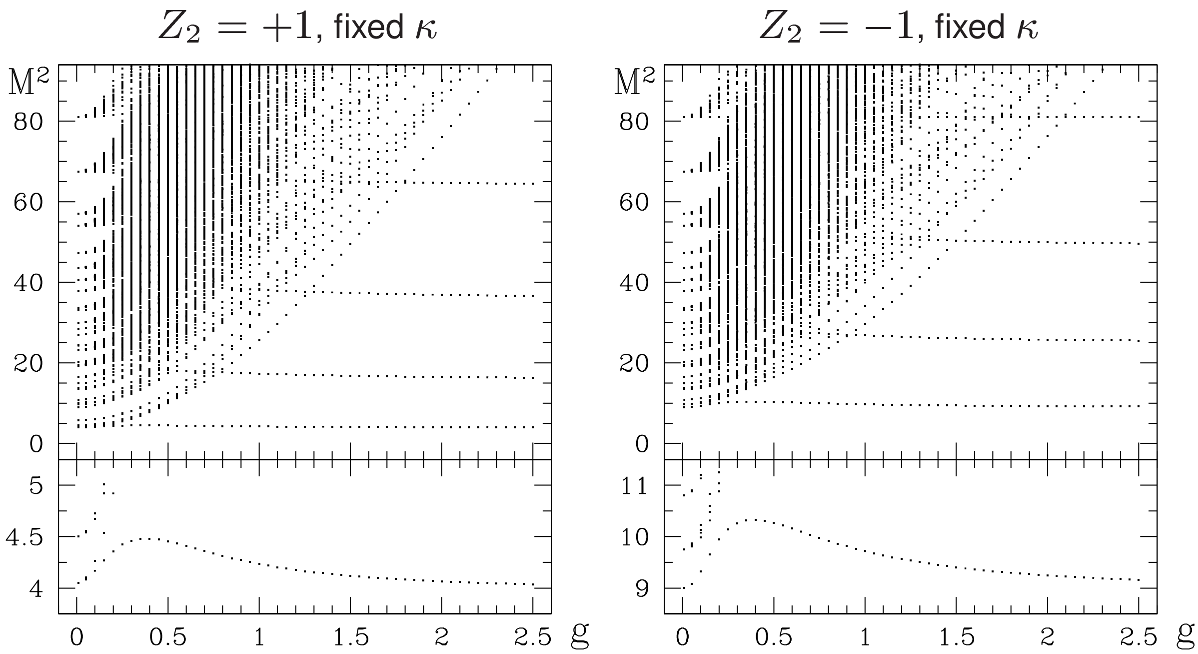


Pure SYM: $2(K-1)$ massless BPS states

Here: mass of some states almost independent of $g \Rightarrow$ “BPS-like” states

We can understand these states at small κ/g, where the CS term is a perturbation

$$Q^- = g\left(Q^-_{SYM} + \frac{\sqrt{\pi}\kappa}{g} Q^-_{CS}\right)$$

We have “BPS-like” bound states with

$$|BPS - like\rangle = |BPS\rangle + \mathcal{O}\left(\frac{\kappa}{g}\right)$$

and (n is number of partons)

$$\bar{M}^2 = \frac{\kappa^2 N_c}{\pi} n^2 + \mathcal{O}\left(\frac{\kappa^3}{g}\right)$$

These states are reflections of the BPS states of the pure SYM theory.

As $g \to \infty$ these are threshold bound states.

Results for 1+1 CS-SYM theory

At $g = 0$ we have only the CS term (a number operator)

$\Rightarrow$ free particle spectrum

Mass of a state with b constituents of mass $m = \kappa$ in DLCQ

$$\frac{M^2(K)}{K} = \frac{\kappa^2}{K - \sum_{i=1}^{b-1} n_i} + \sum_{i=1}^{b-1} \frac{\kappa^2}{n_i}$$

$2, 3, 4, \ldots$-particle continuum at $M^2 = 4, 9, 16, \ldots \times \kappa^2$

$\Longrightarrow$ Complete understanding of small and large coupling regimes!

New duality:

$$\lim_{K\to\infty} M_{BPS-like}(g^2 N_c = 0) = \lim_{g\sqrt{N_c}\to\infty} M_{BPS-like}(g^2 N_c)\,.$$

Example:

$g = 0$: $M^2 = 4$ are 2 free particles of unit mass

$g = \infty$: $M^2 = 4$ single bound state of $M = 2$ (threshold)

Generally:

- CS term generates mass proportional to CS coupling $\Rightarrow$ low-mass states have few partons (QCD-like)
- Compare to pure SYM: low-mass states have many partons (string-like)
- Good convergence!

Structure Functions of Bosons

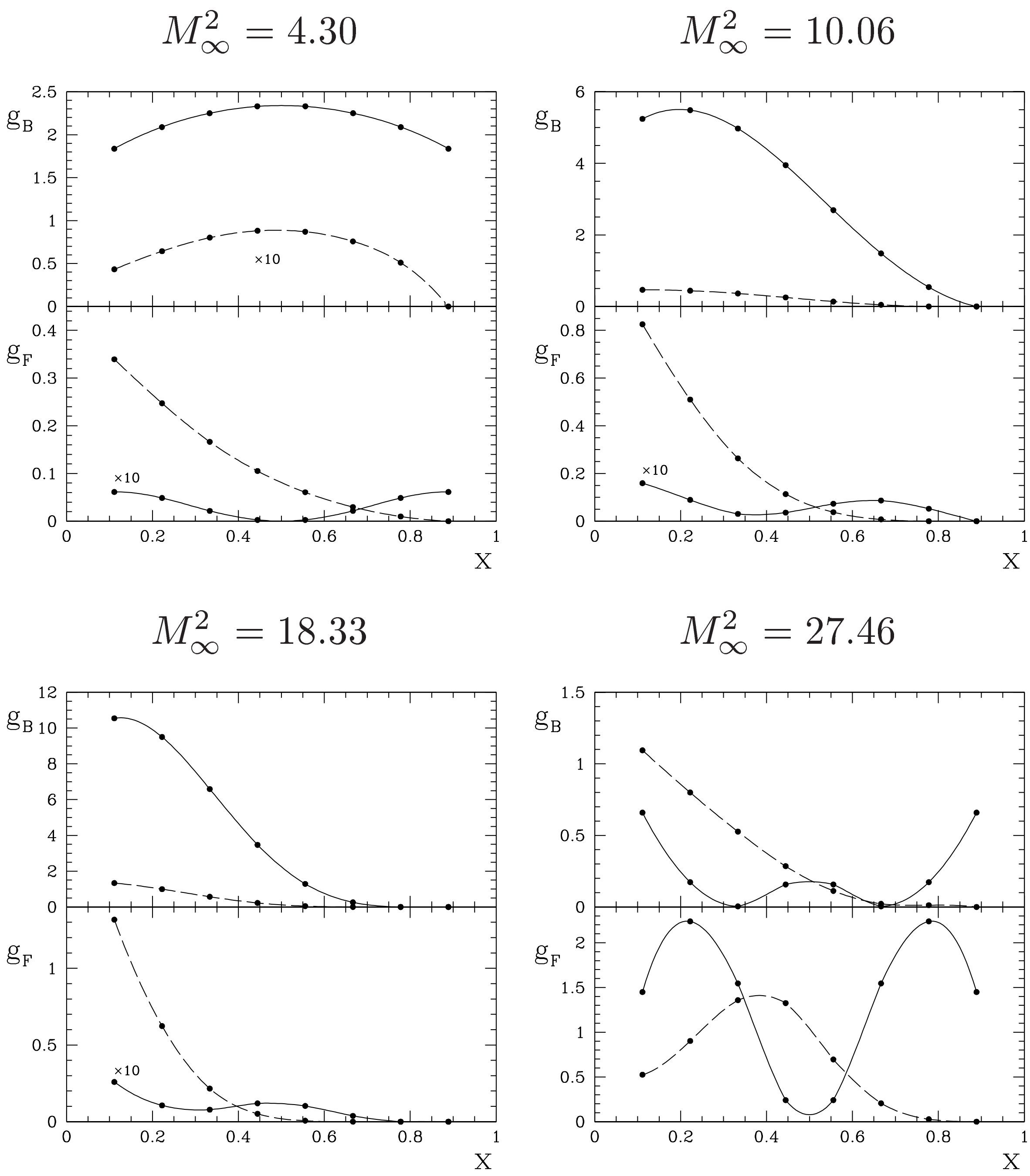


Figure 1: Structure functions of *bosonic* states at K=9. The top of the graphs is bosonic, the bottom is fermionic structure functions. Dashed lines are sea contributions, solid lines are the two-parton (M^2=4.3 and M^2=27.46), three-parton (M^2=10.06) and four-parton (M^2=18.33) sector contributions.

Note: Different scales! The fermionic contributions are suppressed.

SYM-CS theory in 2+1 Dimensions PLB (in print)

Put system in transverse box: $L_\perp > x_\perp > 0$

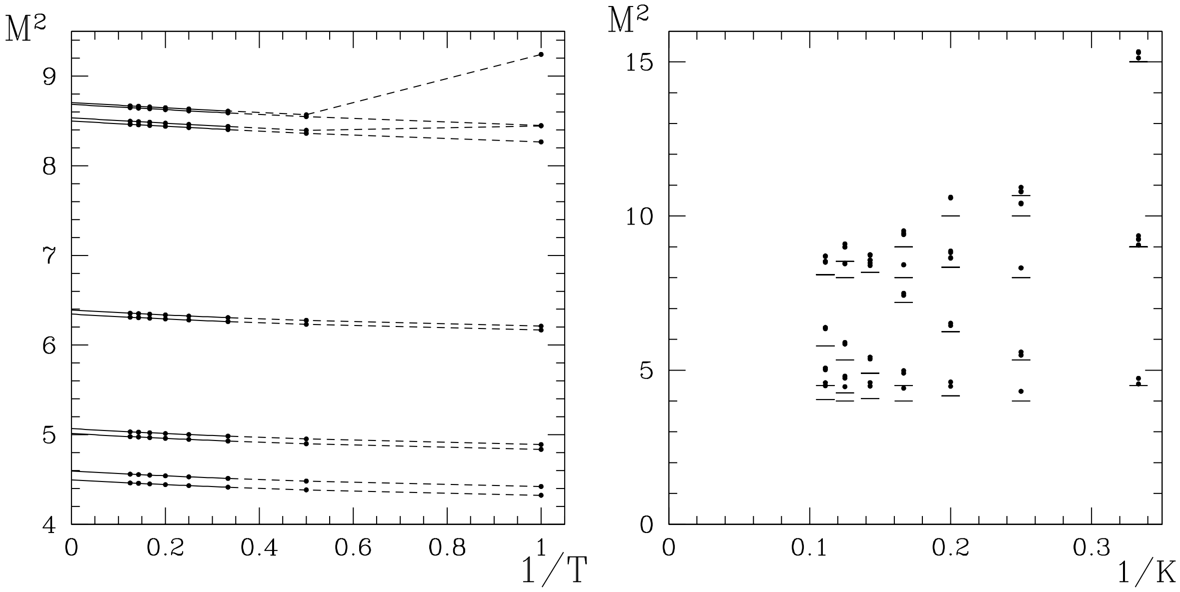
Additional SUSY algebra: $\{Q^+, Q^-\} = -4P_\perp$

Work at $(P_\perp)_{total} = 0 \Longrightarrow \mathcal{N} = (1,1)$ SUSY.

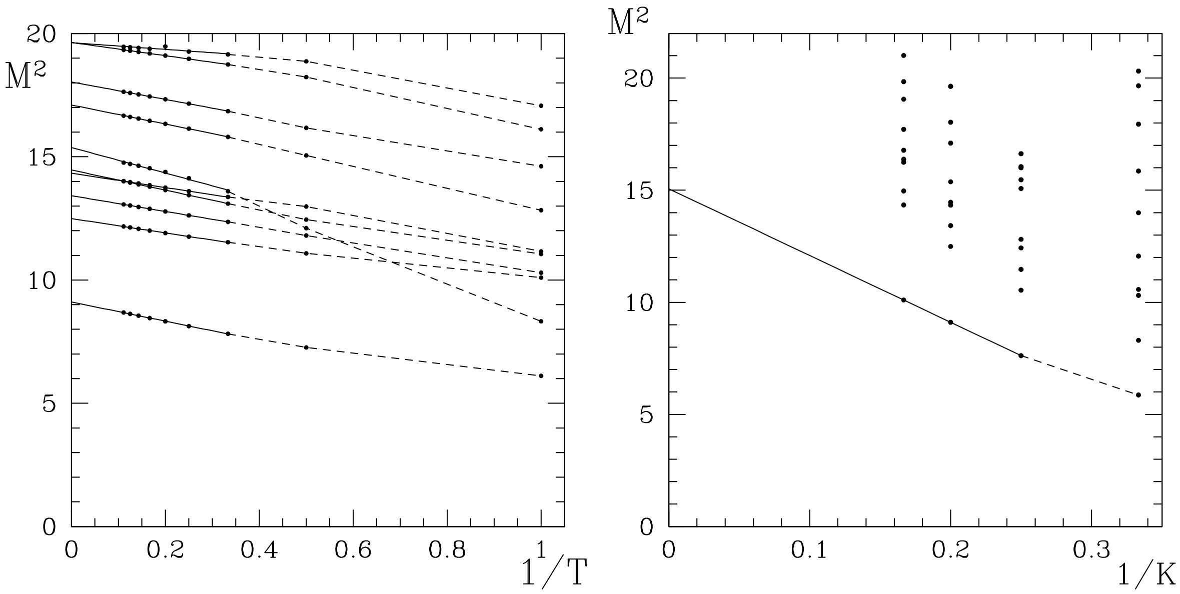
Transverse resolution T yields discretization

$$k_\perp = 2\pi n_\perp / L_\perp; \qquad -T < n_\perp < T$$

$$g = 0.1$$



$$g = 0.5$$



BPS-like states/anomalously light states

Mass spectrum of CS–SYM theory at fixed K and T

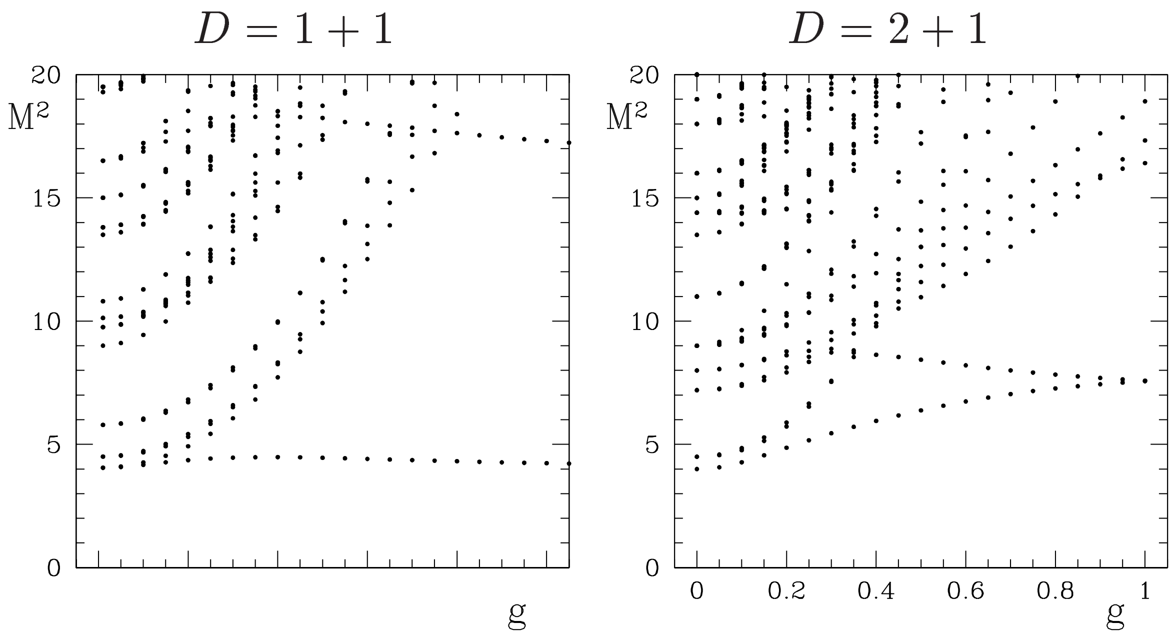


Table: 10 lowest masses at $g = 0.1$ and $g = 0.5$.

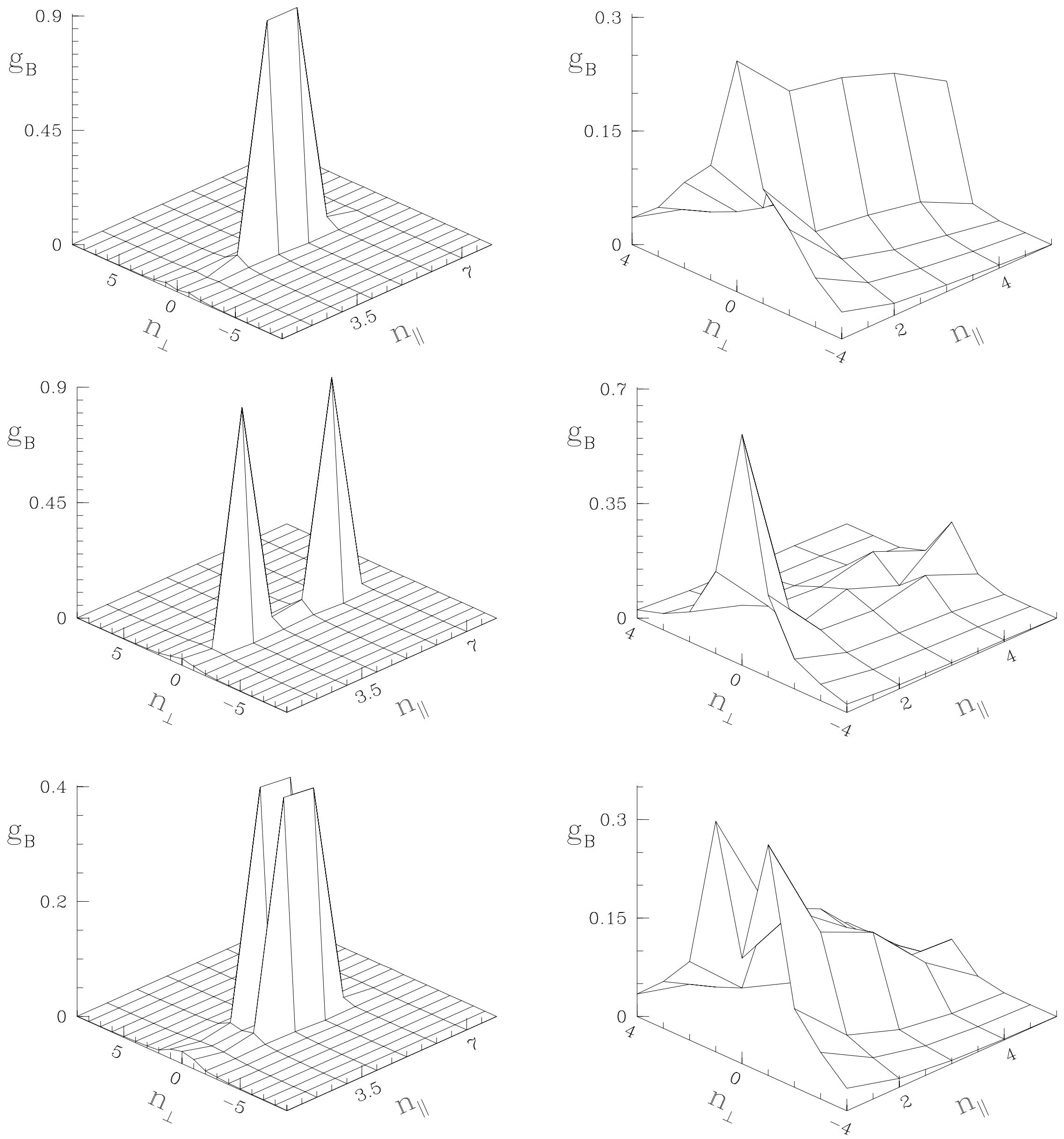
Note: At strong coupling, **large** gap between lowest and next-to-lowest states, at small coupling **no** gap.

g	M^2									
0.1	**4.36**	**4.85**	4.93	7.41	7.47	8.35	8.36	9.34	9.43	9.48
0.5	**8.93**	**13.14**	13.88	14.92	15.10	15.62	15.82	17.38	18.35	19.76

The anomalously low mass state at strong coupling is a reflection of the massless BPS in the underlying pure SYM theory in 2+1 dimensions!

Structure Functions in 2+1 Dimensions

$g = 0.1$ $\qquad\qquad$ $g = 0.5$



$K = 9, T = 9$ $\qquad\qquad$ $K = 6, T = 4$

Large coupling:

- states grow longer
- mixture between KK and longitudinal excitations

Summary

CS term gives masses to constituents $\longrightarrow$ QCD-like structure of states, very different from pure SYM (stringy)

SYM-CS in 1+1

- massless BPS states of SYM theory reflected here by states nearly independent of g which have masses proportional to their parton number

$$\text{Duality:} \left\{ \begin{array}{ll} g = 0 : & \text{continuum of free massive partons} \\ g \to \infty : & \text{threshold bound states} \end{array} \right\} \text{same masses}$$

- distribution functions have valence structure

SYM-CS in 2+1

- approximate BPS states
- anomalously light states might be phenomenologically interesting
- structure functions are solutions of actual gauge theory, not of an ad hoc phenomenological model
- KK modes of models with extra dimensions might be hard to disentangle from regular states