COMPARISON OF BOUND-STATE CALCULATIONS IN LFD AND IFD

M. van Iersel
Vrije Universiteit, Amsterdam

- Introduction
- Model
- Bound-state equations
- Identification of states
- Results
- Summary and conclusions

in collaboration with
B.L.G. Bakker and F. Pijlman

INTRODUCTION

Interested in calculating relativistic bound states in quantum field theoretical approach.

Work done by others: e.g.
LFD :

- S. Głazek et al. — 3+1 Yukawa
- M. Mangin-Brinet et al. — ground states Wick-Cutkosky
- Fuda — NN
- Cooke and Miller — ground state Wick-Cutkosky, box

IFD:

- Fuda — NN / ΠN
- Ding and Darewych — variational method

What is the difference between LFD and IFD?
Calculations done in both forms of dynamics to compare the results.

First calculate bound states (i.e. spectra and wave functions) in a toy model $\rightarrow$ Wick-Cutkosky.

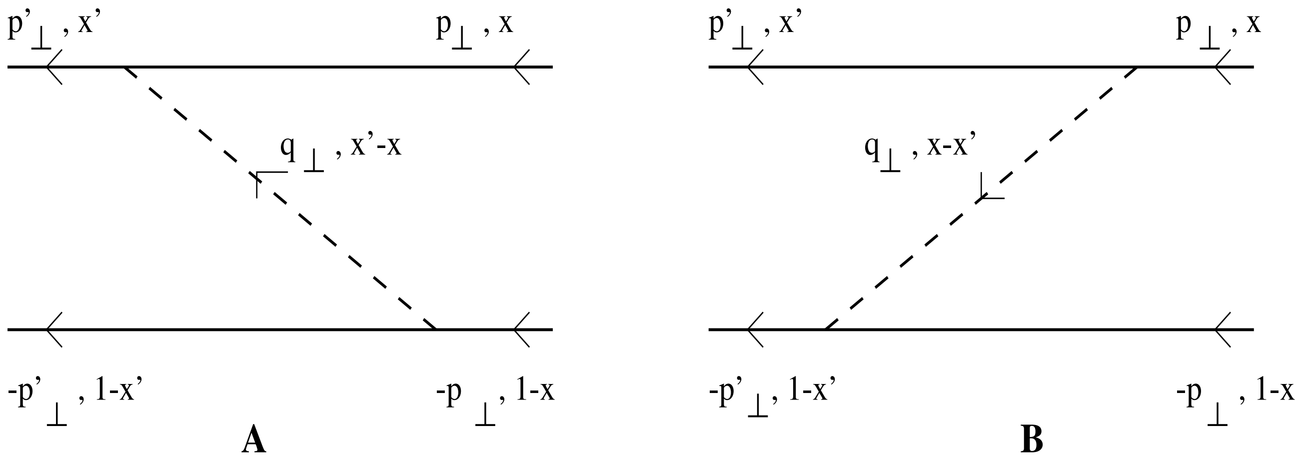
MODEL

Two scalar particles (identical, mass m) exchanging another scalar particle (mass μ).

Lagrangian:

$$\mathcal{L} = \partial_\mu \Phi^\dagger \partial^\mu \Phi - m^2 \Phi^\dagger \Phi + \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{\mu^2}{2} \phi^2 - g \phi \Phi^\dagger \Phi$$

Ladder approximation: first order time-ordered diagrams



No self-energy diagrams

Fock space restricted to 2 and 3 particle sector

BOUND-STATE EQUATIONS

Front form

Hamiltonian: $P^- = \int \mathrm{d}x^- \mathrm{d}^2 x^\perp \; T^{+-}$

Free fields:

$$\phi(x) = \int \frac{\mathrm{d}k^+ \mathrm{d}^2 k^\perp}{(2\pi)^3 \sqrt{2k^+}} \left(a(k) e^{-ikx} + a^\dagger(k) e^{ikx} \right)$$

$$\Phi(x) = \int \frac{\mathrm{d}k^+ \mathrm{d}^2 k^\perp}{(2\pi)^3 \sqrt{2k^+}} \left(b(k) e^{-ikx} + d^\dagger(k) e^{ikx} \right)$$

Schrödinger equation: $\quad 2P^+ P^- |\psi >= M^2 |\psi >, \; \vec{P}_\perp = 0$

Bound-state equation:

$$\left[M^2 - \frac{\vec{p}_\perp^{\,2} + m^2}{x(1-x)} \right] \phi(\vec{p}_\perp, x) = \frac{g^2}{(2\pi)^3} \int \mathrm{d}^2 \vec{p}\,'_\perp \mathrm{d}x' K(\vec{p}_\perp, x; \vec{p}\,'_\perp, x') \phi(\vec{p}\,'_\perp, x')$$

$$K(\vec{p}_\perp, x; \vec{p}\,'_\perp, x') = \frac{1}{\sqrt{x(1-x)x'(1-x')}} \left(\frac{\theta(x'-x)}{2(x'-x)D_a} + \frac{\theta(x-x')}{2(x-x')D_b} \right)$$

$$D_a = M^2 - \frac{\vec{p}_\perp^{\,2} + m^2}{x} - \frac{\vec{p}\,'^{\,2}_\perp + m^2}{1-x'} - \frac{(\vec{p}\,'_\perp - \vec{p}_\perp)^2 + \mu^2}{x'-x}$$

$$D_b = M^2 - \frac{\vec{p}\,'^{\,2}_\perp + m^2}{x'} - \frac{\vec{p}_\perp^{\,2} + m^2}{1-x} - \frac{(\vec{p}_\perp - \vec{p}\,'_\perp)^2 + \mu^2}{x-x'}$$

Instant form

Hamiltonain: $P^0 = \int \mathrm{d}x_1 \mathrm{d}x_2 \mathrm{d}x_3 \; \theta^{00}$

Free fields:

$$\phi(x) = \frac{1}{(2\pi)^{3/2}} \int \frac{\mathrm{d}^3 k}{\sqrt{2\omega}} \left(a(k) e^{-ikx} + a^\dagger(k) e^{ikx} \right)$$

$$\Phi(x) = \frac{1}{(2\pi)^{3/2}} \int \frac{\mathrm{d}^3 p}{\sqrt{2E}} \left(b(p) e^{-ipx} + d^\dagger(p) e^{ipx} \right)$$

Schrödinger equation: $H|\psi >= \mathcal{E}|\psi >$

Bound-state equation:

$$\left[\mathcal{E} - 2\sqrt{\vec{p}^{\,2} + m^2} \right] \tilde{\phi}(\vec{p}) = \frac{g^2}{(2\pi)^3} \int \mathrm{d}^3 p' \tilde{K}(\vec{p}, \vec{p}\,') \tilde{\phi}(\vec{p}\,')$$

$$\tilde{K}(\vec{p}, \vec{p}\,') = \frac{1}{\mathcal{E} - \sqrt{\vec{p}\,'^2 + m^2} - \sqrt{\vec{p}^{\,2} + m^2} - \sqrt{(\vec{p}\,' - \vec{p})^2 + \mu^2}}$$
$$\times \frac{1}{4\sqrt{\vec{p}\,'^2 + m^2}\sqrt{\vec{p}^{\,2} + m^2}\sqrt{(\vec{p}\,' - \vec{p})^2 + \mu^2}}$$

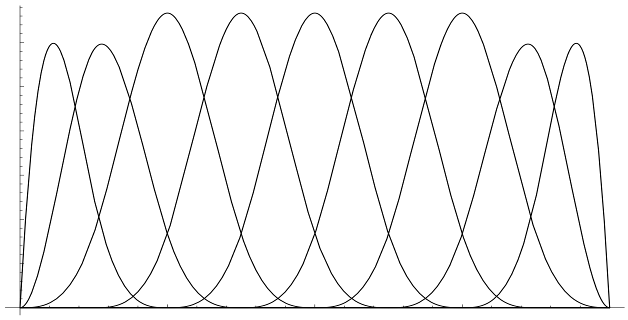
Method of solving

Expand wave functions:

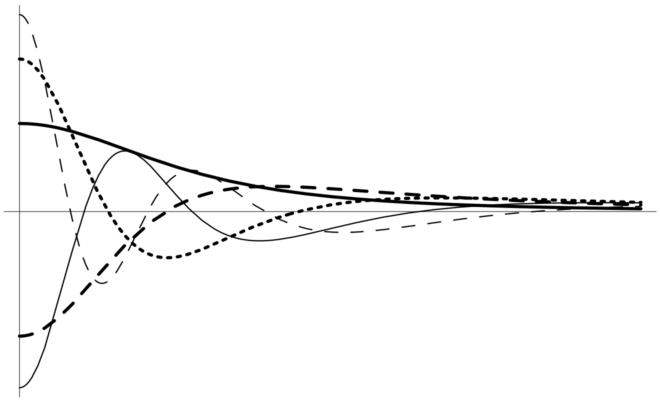
LFD:

$$\phi(\vec{p}_\perp, x) = \sum_h \psi_{\text{spl}}(x)\psi_h^{OW}(|\vec{p}_\perp|)e^{-ih\varphi}$$

B splines $\psi_{\text{spl}(x)}$:



Olsson Wenigers $\psi_h^{OW}(|\vec{p}|)$:



IFD:

$$\tilde{\phi}(\vec{p}) = \sum_{lm} Y_{lm}(\hat{p}) \tilde{\psi}_l^{OW}(p)$$

Integrate out angular dependence

Numerical integration

IDENTIFICATION OF STATES

Calculate bound state mass and wave function and identify these states; i.e. assign angular momentum quantum number.

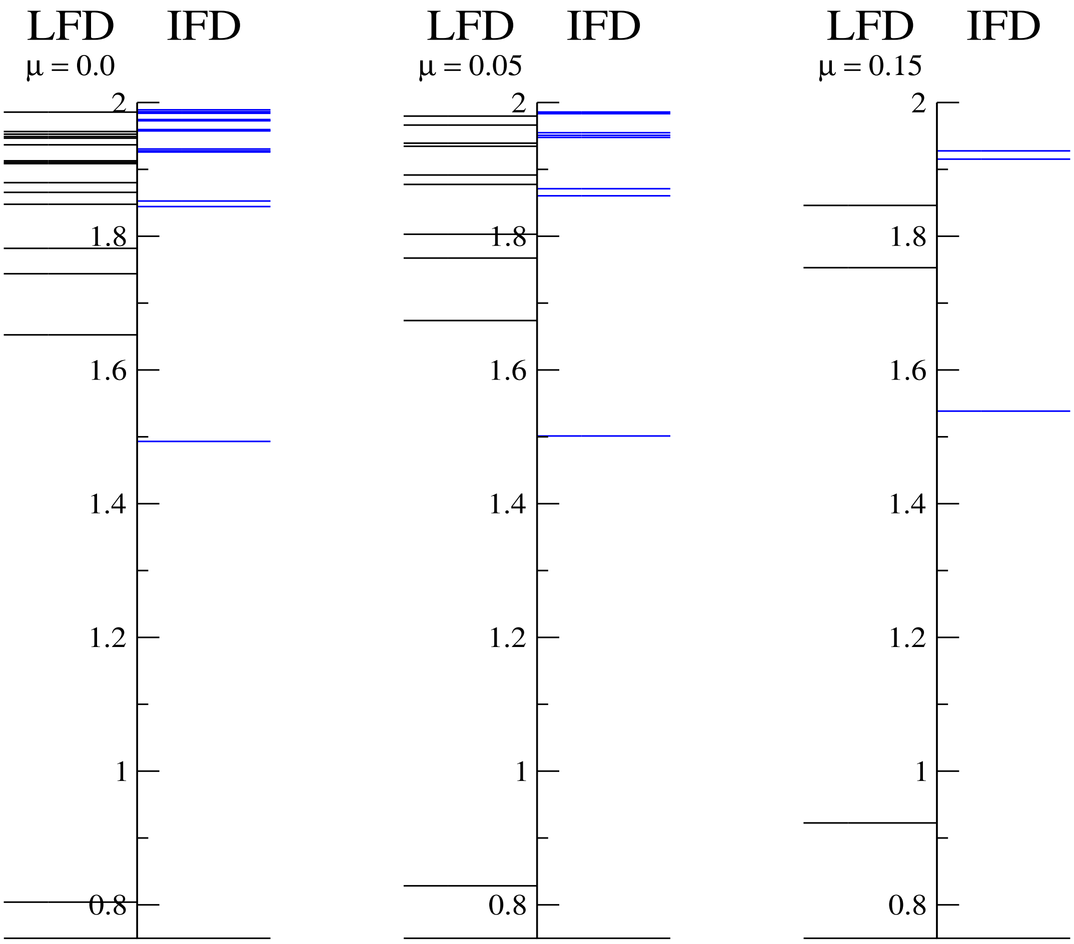
IFD	LFD
mass	mass
spin (l, m)	spin $(l?, h)$
$\vec{L}$ kinematic	$\vec{L}$ dynamic

Approximation:
Change set of variables $(p_\perp, x) \rightarrow (p_\perp, p_z)$ and create some kind of 3-vector.
Terent'ev transformation:

$$x = \frac{\sqrt{\vec{p}^2 + m^2} + p_z}{2\sqrt{\vec{p}^2 + m^2}}$$

RESULTS

$m = 1.0, \;\; g = 17.36, \;\; \alpha = 6$



$l = 0$

$\mu = 0.0$	
LFD	IFD
0.8040 (h = 0)	1.4931
1.7439 (h = 0)	1.8526
1.8657 (h = 0)	1.9304
1.9086 (h = 0)	1.9595
	1.9745

$l = 1$

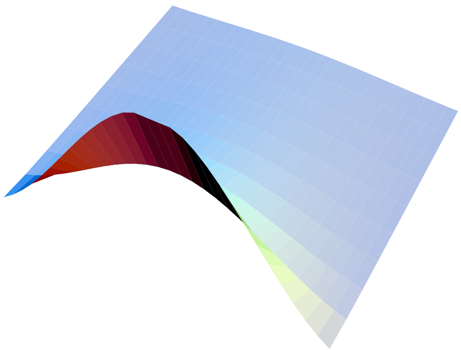
$\mu = 0.0$	
LFD	IFD
1.6525 (h = 0)	1.8443
1.7819 (h = 1)	1.9276
1.8478 (h = 0)	1.9582
1.9128 (h = 1)	1.9733
	1.9862

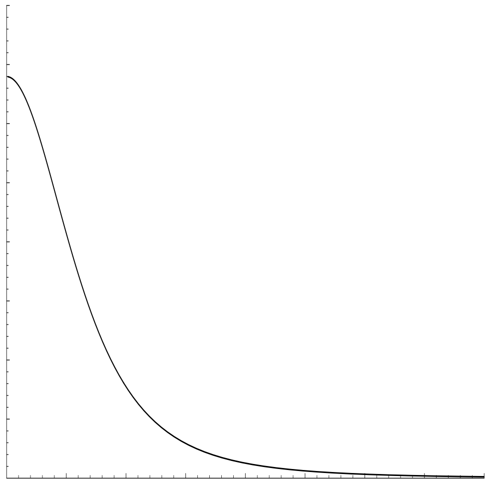
$l = 2$

$\mu = 0.0$	
LFD	IFD
1.8801 (h = 1)	1.9259
1.9104 (h = 2)	1.9575
1.9490 (h = 1)	1.9726
	1.9839

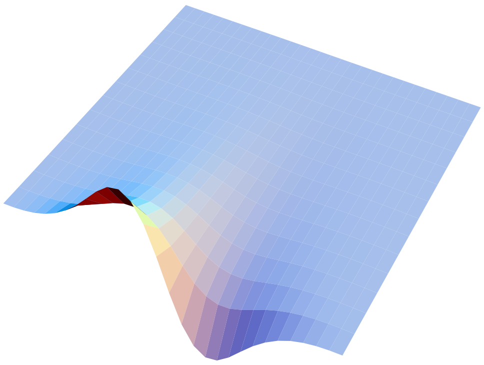
Wave functions

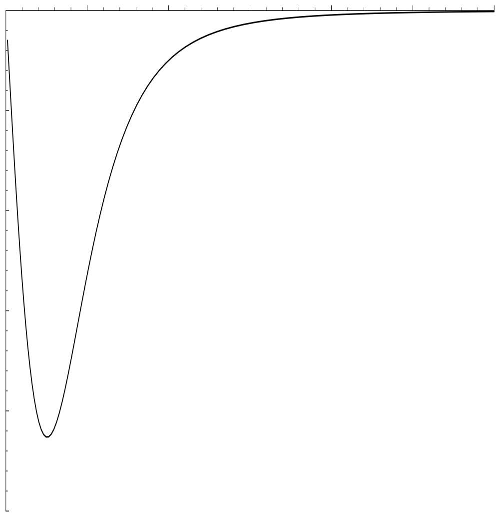
Ground state: $\mu = 0$, $l = 0$, helicity = 0
Mass = 0.8040





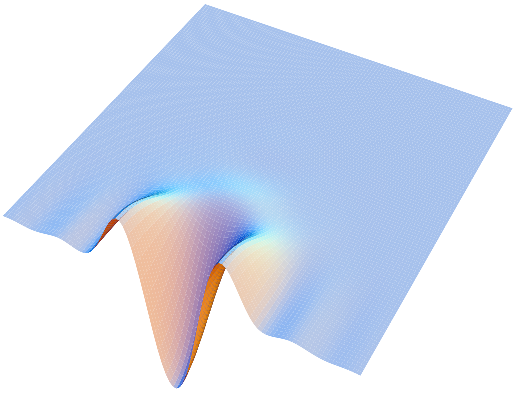
First excited state: $\mu = 0$, $l = 1$, helicity = 0
Mass = 1.6525

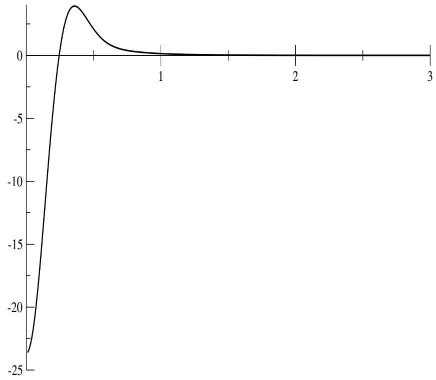


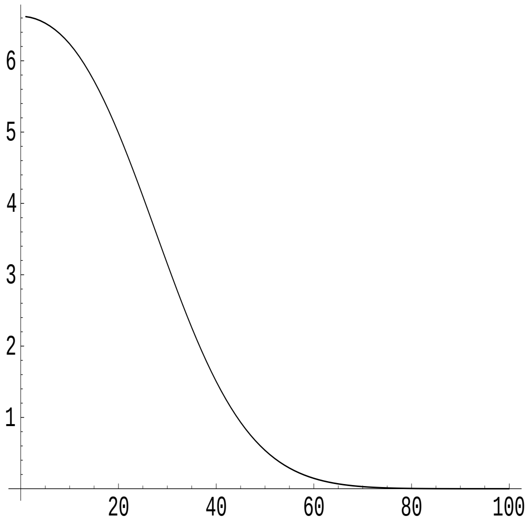


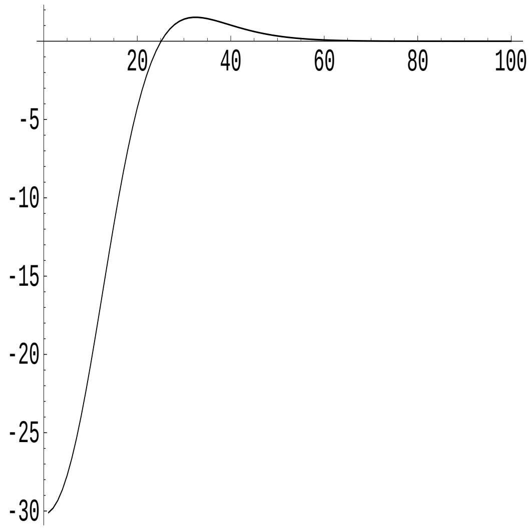
Higher excited state: $\mu = 0$, $l = 0$, helicity = 0

Mass = 1.9086






1
2
3
4
5
6
20
40
60
80
100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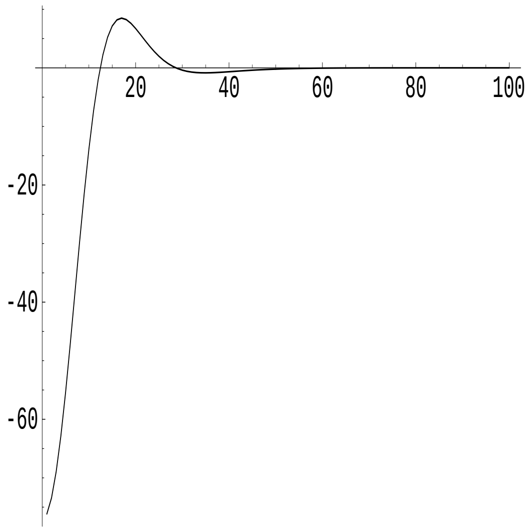
-5
-10
-15
-20
-25
-30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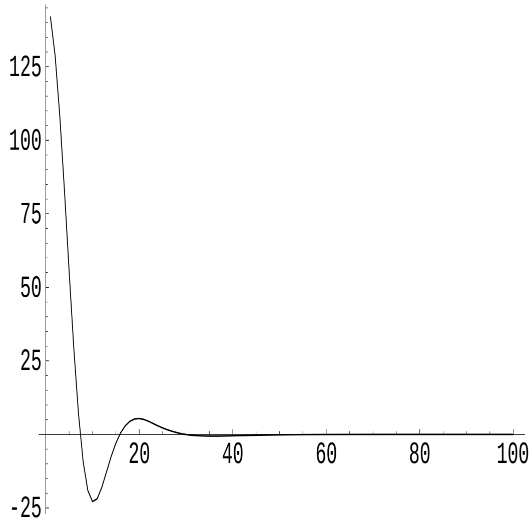
60
80
100
-20
-40
-60


125
100
75
50
25
20
40
60
80
100
-25

SUMMARY AND CONCLUSIONS

Calculate spectra in both LFD and IFD.

States in LFD are deeper bound compared to IFD.

Using the Terent'ev approximation we were able to identify the states in LFD.